Summary

-Topics in Financial Economics-

Basic Concepts

It is not true that we need financial markets just because people are interested in speculation and making profits. We need financial markets because we want to **finance** other great ideas. Everything in financial markets is about risk and return.

Markets **allocate resources** effectively. This happens through the prices, they are signals that direct economic resources to their best use. Prices tell us how scarce things are.

There are mainly two types of markets;
- **Product market**, for manufactured goods and services
- **Factor market**, for labor and capital. The financial market is one important part of this market

People should ask themselves whether their use of a product is more beneficial or not. Prices give us a **signal** of how to use it. Governments can also send signals of values of products by determining its price. In communist states this happens to a very high degree for example.

An **asset** is any possession that has value in an exchange.
- Tangible assets are assets whose value depend on particular physical properties.
- Intangible assets are legal claims to secure some future benefits.
  - Financial asset, like financial instruments or securities, are intangible assets.

Issuers of the financial asset are the entities that agree to make future cash payments. **Investors** are the owners of the financial asset. They can receive cash payments in the future.

- **Debt instruments** are instruments where the issuer needs to repay a fixed amount of money during a specified period. Bonds fall under this definition.
- **Equity instruments** are instruments where the issuer needs to repay an amount based on earnings over indefinite period after holders of debt instruments have been paid. Shares fall under this definition.

Some securities fall into both categories, these are preferred stocks or convertible bonds.

In financial markets, the **return** is generated by some form of tangible assets. Financial markets exist to help other markets. It does not create anything itself. However, financial and tangible assets are expected to generate future cash flow. Also, returns always have some uncertainty;
- Purchasing power risk (or inflation risk)
- Default risk
- Exchange rate risk
- Risk in the amounts to be paid
- Risk in the timing of payments

Money, like crypto or fiat has value based on reputation and **trust**. Digital money has value because people trust that others will accept it.

Financial assets have two purposes;
- Transfer funds from surplus parties to those who need to invest in tangible assets
- Transfer funds in such a way as to redistribute risks, this is portfolio management.

Financial institutions provide **services** related to one or more of the following;
- Transforming financial assets into more preferable assets
- Exchanging financial assets on behalf of customers
- Exchanging financial assets for their own accounts

- Assisting in the creation of financial assets for their customers and then selling those assets to other market participants
- Providing investment advice to other market participants
- Managing the portfolios of other market participants

Liability Type	Amount of Cash Outlay	Timing of Cash Outlay
Type I	Known	Known
Type II	Known	Uncertain
Type III	Uncertain	Known
Type IV	Uncertain	Uncertain

The nature of liabilities of financial institutions is displayed in the table above, these are some corresponding examples;
Type I: Fixed-term deposits with fixed interest rate
Type II: Life insurance policy
Type III: Certificates of deposit with floating interest rate
Type IV: Automobile and home insurance

There are, however, some **liquidity concerns**. Financial institutions must have sufficient cash to satisfy its obligations. There is also some uncertainty about the timing and/or the amount of cash outlay.

Banks are highly leveraged institutions, there is **insolvency risk**. The Basel Committee decided that more banking supervision was necessary.
- **Basel I** came with risk-based capital requirements. The capital accord of 1988 was issued. Its objective was to establish minimum capital standards against credit risk. It was amended several times.
- **Basel II** was a forward looking approach to capital supervision. Pillar I: minimum risk-based capital requirements. Pillar II: supervisory review of capital adequacy. Pillar III: Market discipline through public disclosure.
- **Basel III** was a stricter version of Basel II and introduced additional capital buffers. Important concepts were liquidity requirements and leverage ratio. There were additional proposals for systemically important banks.

Capital requirements means that you have to prepare a payment when it is needed. You need to be able to pay this certain amount of money. So, some money has be liquid and can therefore not be invested. This is a trade-off for the banks. Definitions of capital are getting tighter over years.

There are risk-based capital guidelines for two tiers. Tier 1 is **core capital**, these are available for immediate payments, like emergencies. This capital has to be highly liquid and includes common stock, certain preferred stock and minority interest. Tier 2 is **supplementary capital**, this includes loan-loss reserve, certain preferred stock, perpetual debt, hybrid capital instruments, and subordinated debt. These can not be sold in one day, but in a week or month. It has higher returns, but is less liquid.

Risk Weight	Example of Assets Included
0%	U.S. Treasury securities Mortgage-backed securities issued by the Government National Mortgage Association
20%	Municipal general obligation bonds Mortgage-backed securities issued by the Federal Home Loan Mortgage Corporation or the Federal National Mortgage Association
50%	Municipal revenue bonds Residential mortgages
100%	Commercial loans and commercial mortgages LDC loans Corporate bonds

The table above shows the risk weight of the relevant examples of assets. How much a bank has to set aside is dependent on how much **risk** a bank is taking. If the bank takes more risk, it needs better insurance. When applied to the financial assets of a bank, it looks a bit like the table below.

Asset	Book value	Risk Weight	Product
U.S. Treasure	100	0%	0
Municipals bonds	100	20%	20
Residential mortgages	500	50%	250
Commercial loans	300	100%	300
Risk-weighted assets			570

Capital requirements increase with **bank loans' probability of default** and **loss given default**. These are both likely to increase during downturns. They exacerbate the negative impact of recessions on banks' supply of credit and on the economy as whole.

Interest Rates

An **interest rate** is the price paid by a borrower (or debtor) to a lender (or creditor) for the use of resources during some interval.

Interest rates are interesting because they have an effect on the **real domestic economy**. Changes in short interest rates may influence expenditures. Also, interest rates influence real investment decisions. On top of this, they affect long maturity bonds and stock markets.

Interest rates could have an effect on **expectations** and international **capital flows**. A rise in short interest rates leads to lower inflation expectation and thus lower future short rates and lower long rates. Unexpected changes in short rates may influence capital flows and the exchange rate.

Interest rate and bond prices can be used as a **test-bed** for market efficiency and other theories. Bond prices can be used to test EMH and Shiller's variance bounds tests can also be used for bond markets with regard to perfect foresight yield vs actual yield.

$$P = \frac{CF_1}{(1+r)} + \frac{CF_2}{(1+r)^2} + \ldots + \frac{CF_N}{(1+r)^N}$$

Cash flow:			
CF_1 = \$50	CF_2 = \$50	CF_3 = \$50	CF_4 = \$1,050
Appropriate Discount Rate (%)			Price (\$)
4			1,036.30
5			1,000.00
6			965.35
7			932.26
8			900.64
9			870.41

	Number of years to maturity			
Discount Rate (%)	4	10	15	20
4%	\$1,036.30	\$1,081.11	\$1,111.18	\$1,135.90
5	1,000.00	1,000.00	1,000.00	1,000.00
6	965.35	926.40	902.88	885.30
7	932.26	859.53	817.84	788.12
8	900.64	798.70	743.22	705.46
9	870.41	743.29	677.57	634.86
10	841.51	692.77	619.70	574.32
11	813.85	646.65	568.55	522.20
12	787.39	604.48	523.24	477.14
13	762.04	565.90	483.01	438.02
14	737.77	530.55	447.20	403.92

Fisher's Classical Approach

In this approach there are individuals, firms, a market and projects. This is the **general frame work**. Capital is provided by households. They provide capital because we want to save some money. Not everything that is earned is spent.

The **supply** of savings is determined by several factors. One of which is one's **income**. The more money one makes, the more one will save. If I barely make a living, I cannot save. There is also **time preference**. How much are you willing to save, are you patient enough. Another factor determining how much I am willing to save is the interest rate. If this is high, I get a better **reward for saving**. This will lead to the consumer being more likely to save or to invest. This means there is more supply. High interest rates discourage consumption.

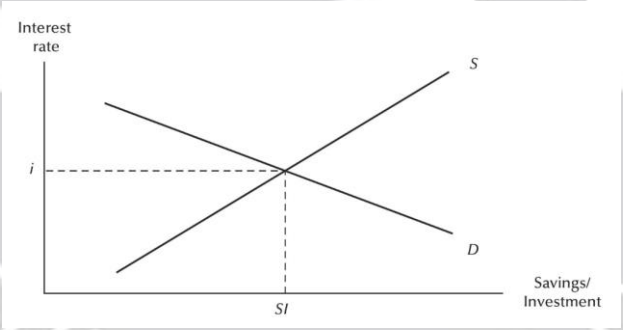
The **demand** for capital comes from **firms**. Firms see profitable investments. That is what they want the money for. The demand depends on the **marginal productivity** of capital. How much can I make from $1? The **rate of interest** is the cost of capital for the firms. A low interest rate means firms are more likely to borrow. When interest rate is low, people or in this case firms, borrow more.

Fisher's law includes nominal rate of interest (i), real rate of interest (r) and premium for expected inflation (p).

$$(1 + i) = (1 + r)(1 + p)$$

$$i \approx r + p$$

This supply and demand for capital comes together in an **equilibrium**. It leads to an equilibrium interest rate.



If households become more patient, the supply curve will shift to the right. What if a new technology has been developed? Then the marginal productivity increases. The demand curve will move to the right as well. A firm can make more per dollar, so the demand will rise and firms are willing to pay a higher interest rate because the marginal productivity is higher.

Loanable Funds Theory

This theory is very similar to Fisher's classical approach. In the Fisher's approach, supply comes from households. But firms also provide capital to the market. The government raises tax, these taxes can also serve as supply to the financial markets. So supply and demand both become **broader**. More actors can be supply and demand. It is a more general idea.

This theory states that the general level of interest rate is determined by the complex interaction of two forces.

- The **first force** is the **demand of funds** by firms, governments and households. These actors carry out economic activities with the funds. This demand is negatively related to interest rates. An increase in interest rate will make projects less profitable, and consumption more costly. So if interest rates increase, the demand will decrease, and vice versa.
- The **second force** is the total **supply of funds** by firms, governments and households. Supply is positively related to interest rates. With rising interest rates, firms and individuals save and lend more, and banks are more eager to extend more loans. So if interest rates increase, the supply will increase, and vice versa.

Shifts of demand and supply have multiple causes. They are changes in money supply or government deficits. But also, changes in preferences by households and new investment opportunities for firms.

The Liquidity Preference Theory

In this approach, the essential object we are concerned with is **liquid money**. Your savings or cash, is the demand we are concerned with. There are two financial assets; one does not provide interest but is highly liquid, the other is not liquid but provides better interest. These are **money** and **bonds**. This theory is more about the real money, for the money market.

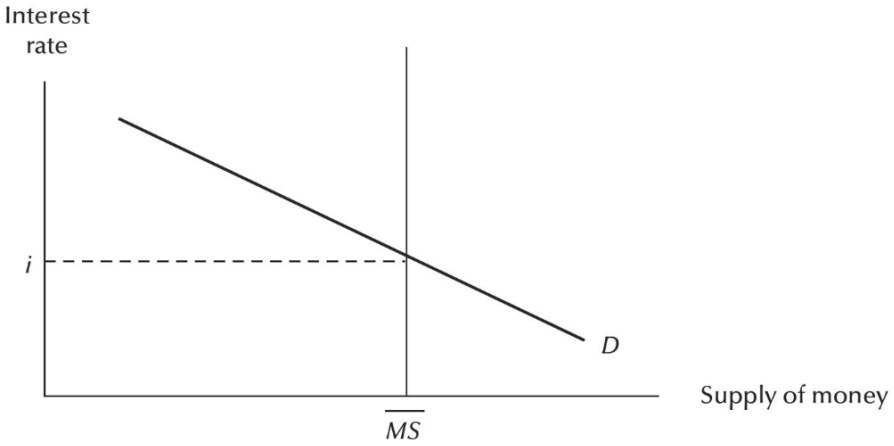
The demand comes from individual households or financial institutions. We want to satisfy these three demands;
- Bonds are very secure and provide better interest than liquid money, so why do people still keep cash? This is because cash is liquid. People need money to buy stuff, this is **transactions demand**. One cannot easily take out the money from a bond.
- We can not perfectly foresee the future and therefore need some money that is highly liquid, this is **precautionary demand**.
- Also, one might think the current market is not too good, you are waiting for the right moment and have the money to invest then, this is **speculative demand**.

Only the central bank supplies money. It has full control of the **supply**. Supply of central bank does not reply to the interest rate. Their supply is therefore a straight line. It is unresponsive. The focus is to intervene in the economy.

The **demand** curves slopes down in this theory because when the interest rate is high, most people invest more and hold less cash. Alternatively, they hold more money when the interest rate is low.

- Liquidity effect, if increasing, causes the interest rate to rise
- Income effect, if increasing, causes the interest rate to rise
- Price expectations effect, causes the interest rate to rise
- This results in a **net effect**

When the economy is slowing down, more money will be **pumped** into the system. That means the supply curve moves goes to the right. The immediate reaction is a decrease of the interest rate. People will spend more, meaning the consumption increases. Then, firms will hire more people. This is the intention of the money **supply policy**. Employment goes up and everyone has more money, this leads to inflation. Inflation will push up demand for money even further, because one needs more money for the same consumption. In the long run, it is maybe not such a good theory.



In the market, there is more than just one interest rate. The structure between the maturity and the interest rate is the **term structure of interest rates**. Expectation in economy is very important, this serves the term structure of interest rates.

A **bond** is defined by the term to maturity, the principal (how much you receive in the end) and the coupon (small payments that are fixed). This defines a bond. An interest rate can be fixed or floating. Bonds have different maturities, principals and coupons, how can we determine them? There is one easy way, which is to calculate the **yield** of the bond. It is the average return of a bond over a lifetime.

To do this, future payments have to be discounted. The **yield to maturity** consists of coupon interest, capital gain or loss and is calculated as follows. You can get the principal from the market, as well as the coupon. So the only unknown is the yield. This can be calculated. With this yield, we can **compare bonds**. It is calculated by trial and error.

$$P_t = \frac{C}{(1+y_t)} + \frac{C}{(1+y_t)^2} + \ldots + \frac{C+M}{(1+y_t)^n}$$

The yield to maturity is **artificial**. You cannot take it as a real interest rate. It is really only to compare investments. It tells you the **average return**. The spot rates are the real interest rates if you go to the bank today.

Long term bonds pay coupon rates. This coupon payments mess up the calculation. To calculate the missing yields (spot rates), we use the **bootstrapping method**. The rates need to be calculated because they are often not directly available.

The following theories regard ideas about the term structure.

Maturity	Yield to Maturity	Theoretical Spot Rate
0.50	0.0800	0.08000
1.00	0.0830	0.08300
1.50	0.0890	0.08930
2.00	0.0920	0.09247
2.50	0.0940	0.09468
3.00	0.0970	0.09787
3.50	0.1000	0.10129
4.00	0.1040	0.10592

Table: Yield to maturity to spot rates.

$z_1 = 0.04$ and $z_2 = 0.0415$

The present value of the 1.5-year treasury coupon security:

$$\frac{4.25}{(1+0.04)^1} + \frac{4.25}{(1+0.0415)^2} + \frac{104.25}{(1+z_3)^3} = 99.45$$

Solve for z_3, we have

$$z_3 = 0.04465$$

thus annualized spot rate for 1.5-year zero coupon treasury is: 0.0893

With z_3, we can do the same for two-year spot rate

$$\frac{4.5}{(1+z_1)^1} + \frac{4.5}{(1+z_2)^2} + \frac{4.5}{(1+z_3)^3} + \frac{104.5}{(1+z_4)^4} = 99.64$$

This is called bootstrapping

Pure expectation theory

This theory states that the long-term interest rate only depends on the **short-term expected interest rate**. People do not care about a term. Whether this term is long or short. People have no preference for long or short-term, it is only about short term expected interest rate.

Thus, the interest rate on a long-term bond will equal an average of the short-term interest rates that people expect to occur over the life of the long-term bond. Buyers of bonds do not prefer bonds of one maturity over another. They will not hold any quantity of a bond if its expected return is less than that of another bond with a different maturity.

It is further assumed that there are no transaction costs and that bonds are perfect substitutes of each other.

Liquidity premium theory

The key-observation here is that long-term bonds entail greater **market risk** than short-term securities do. Market risk is the risk of fluctuation in the price of the security due to interest rate changes. Investors may have to sell their assets **prior to maturity**, exposing themselves to the possibility of losses as interest rate and thus market price change.

If bond buyers are risk averse, they must be **compensated** with a **term premium**.

$$R_{t,t+n} = R_{t,t+n-1} + TP$$

This theory states that the term premium (*TP*) is positive and increases with the term to maturity. A consequence is that a bond with longer maturity provides a higher yield.

$$R_{t,t+n} = \frac{r_t + E_t(r_{t+1}) + E_t(r_{t+2}) + \ldots + E_t(r_{t+n-1})}{n} + TP_n$$

Market segmentation theory

This theory says that the long-term market and short-term bond market are **separate markets**. In a way, we assume there are market frictions. Investors easily move around each other. If I am in a long-term market, anything in the short-term market is irrelevant to me. Interest rates do not connect with each other in between these two markets. Companies sometimes face **constraints**, they are forced to borrow for some certain term. For them, the short en long-term is thus separate. Markets are not perfect anymore. There is no perfect competition anymore. In this theory, one can no longer freely move between markets, because they are not perfect.

Interest rates then depend on the **supply** and **demand** in each market. The short-term interest rates depend on the short-term supply for fund and short-term demand for fund. The long-term interest rates depend on the long-term supply for fund and long-term demand for fund.

Short-term securities provide **liquidity** and price stability. Long-term securities provide stable **income**.

Firms and individuals want to **match** the maturities of their assets with the maturities of their liabilities. Firms borrowing to finance inventories prefer short-term loads. Families buying homes prefer long-term fixed rate mortgages.

Preferred habitat theory

It is a hybrid model. This theory says that some people have a **particular habit**, a preferred action. Borrowers and lenders have strong preferences for particular maturities. People deviate from their preferred maturities if expected additional returns from deviation become large enough. This connects different markets again.

Institutions move among different markets.

The main idea is to test whether our expectations are **rational**. If our expectations are systematically wrong, then people should be worried. Markets would not be efficient and there would be dangers of exploitation. Some make use of the irrationality of market participants. It is important to find this out. There are many ways.
The **Expectation Hypothesis** implies that the expected return on different maturity bonds, over any holding period, should be equalised. For example, the return on holding any *n*-year zero coupon over a one-year horizon should equal the known yield on a risk-less one-year bond. If the yield on a ten-year bond is currently above the one year bond, does this mean a violation of the EH?

$P_t^{(n)}$: the price of a zero coupon with maturity of *n* at time *t*.
r_t : the spot interest rate from *t* to *t* + 1.
$R_t^{(n)}$: the long rate at time *t* for *n* years.
T_t^n : the risk premium

$$ln\left(P_t^{(n)}\right) = ln\left(\frac{1}{\left(1+R_t^{(n)}\right)^n}\right) = -nR_t^{(n)}$$

Expected holding period return is defined as:

$$E_t(h_{t+1}^{(n)}) \equiv E_t\left[ln\left(P_{t+1}^{(n-1)}\right) - ln\left(P_t^{(n)}\right)\right] = nR_t^{(n)} - (n-1)R_{t+1}^{(n-1)} = r_t + T_t$$

Statistic testing requires variables are stationary. However, prices are not stationary and sometimes returns are neither.
Model 1, change-in-long-rate equation, with:

$$(n-1)\left[R_t^{(n)} - E_t(R_{t+1}^{(n-1)})\right] = \beta(r_t - R_t^{(n)}) + T_t^n$$

Model 2, long rate are a weighted average of future short rates:

$$R_t^{(n)} - r_t = E_t\left[\frac{1}{n}\sum_{i=0}^{n-1}(r_{t+i} - r_t)\right] + E_t\left[\frac{1}{n}\sum_{i=0}^{n-1}T_{t+i}^{(n-i)}\right]$$

Future-short-rate equation (subtracting r_t from both sides of the above equation):

$$R_t^{(n)} - r_t = E_t\left[\frac{1}{n}\sum_{i=0}^{n-1}(r_{t+i} - r_t)\right] + E_t\left[\frac{1}{n}\sum_{i=0}^{n-1}T_{t+i}^{(n-i)}\right]$$

A test of EH is the regression:

$$E_t\left[\frac{1}{n}\sum_{i=0}^{n-1}(r_{t+i} - r_t)\right] = \alpha + \beta(R_t^{(n)} - r_t) + \gamma\Omega_t + \frac{1}{n}\sum_{i=0}^{n-1}\eta_i$$

Where $n_i = r_{t+i} - E_t(r_{t+i})$ is the forecast error. The constant risk premium is constant.

Asset Pricing

Investors need asset pricing models to make **investment decisions** and **determine risk**. From a practitioners point of view, one needs to know asset prices. A lot of models have been developed for this.

Risk-neutral asset pricing is purely based on the idea that you have two payment streams, one now and one in the future, these have to be priced the same. It is about duplicating future payment streams. The pricing happens relative to the benchmark.

General equilibrium models are about the simple idea of supply and demand. Prices will adjust through market clearing. These models try to determine factors that drive supply and demand.

A **stock** is ownership in a company, the owner is entitled to dividend payments. But what drives the supply and demand of stock prices? The supply and demand of stock prices are based on risk and return. Finance is essentially the same, it is moderating between **risk** and **return** and attempts to find an equilibrium.

The CAPM is a **preference based equilibrium model**. Asset returns are assumed to be normal, which means that the future returns are normally distributed. As said, investors care about risk and return, μ and σ. Some predictions of the CAPM;

- Everyone holds the same risky portfolio: the **market portfolio**.
- Depending on risk aversion, people mix more or less with risk-less investment.
- The expected excess return (μ_i - r_f) of any portfolio *i* is **linear** in the market portfolio's excess return (μ_m - r_f).

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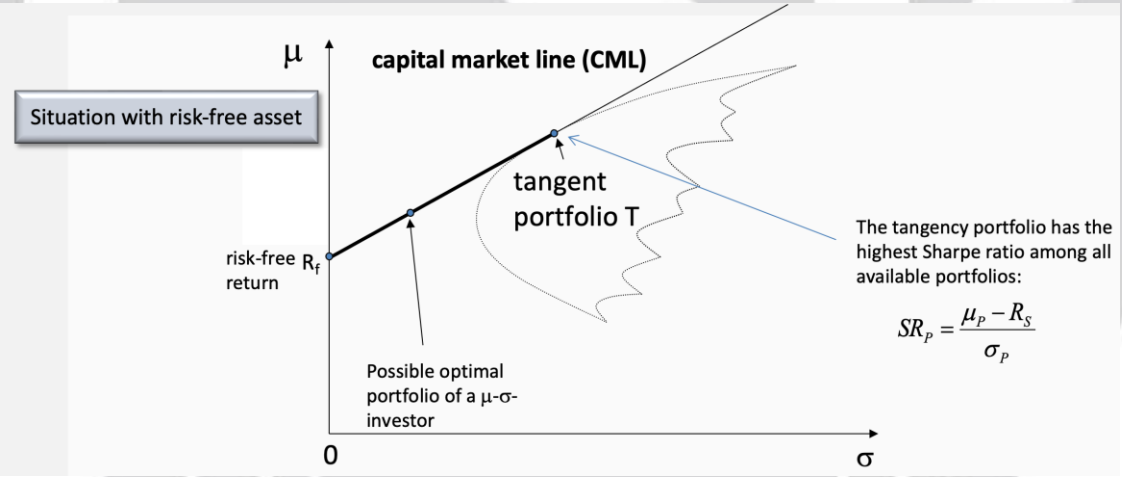


Tobin separation theorem states that the structure of the risky part of the optimal portfolio does not depend on the degree of risk aversion of the investor. The investor's **risk aversion** only determines how he splits up his budget on the risky (tangency) portfolio and the risk-free asset.

If all investors are investors that look at the difference between risk and return, the market portfolio has to have the same structure as the **tangent portfolio**. Therefore, the only risk factor that is relevant is how much risk is added to this market portfolio. β is the standardised covariance and the only relevant risk-factor.

CAPM is fine, but really, people care more about **consumption**. Return is great but if inflation is higher, it becomes less relevant. There is some form of consumption risk. Assets that allow for additional consumption in times when consumption level is low are valued more highly. Prices of such assets are bid up and thus returns are low.

This effect moves asset prices when the majority of investors values an asset highly. Think of **recessions**, most people consume less because jobs are insecure in recessions. But also **inflation** affects everyone because people consume more or less the same consumption bundles.

This line of reasoning leads to the **Consumption CAPM** (CCAPM), or the **Intertemporal CAPM** (ICAPM). If an asset or portfolio is correlated with future state variables that affect investment opportunities (ICAPM) or aggregate consumption (CCAPM), we can use this asset to **hedge** against **bad opportunities** or low consumption in the future. It extends the CAPM to:

$$\mu_i - r_f = \beta_{i,M} \cdot (\mu_M - r_f) + \beta_{i,1} \cdot \lambda_1 + \cdots + \beta_{i,n} \cdot \lambda_n$$

CAPM part

Additional risk premia related to future investment / consumption (can be proxied by mimicking portfolios)

The **Arbitrage Pricing Theory** (APT) states that any type of (macroeconomic) factor could be an additional risk factor. Here, markets are **efficient** and thus prices are right. Any (expected) return of an asset has to stem from all **risk factors** combined, λ's can be anything.

Arbitrage cannot happen in an efficient market. This also means that all prices must be right. In that case, the market tells us prices of assets. Expected returns must therefore come from some type of **risk**. Risk cannot just be measured by standard deviation. Anything that influences the return has to be a risk factor in and of itself.

Until the mid 70's, empirical checks of the CAPM were largely considered to be successful. Then, empirical evidence against the CAPM came up. Also, **critique** on theoretical grounds was put forward. The market portfolio is not observable (has to include everything) and the CAPM's prediction is a mathematical tautology. The predictions and assumptions of the CAPM are not met on the micro-level.

Form portfolio's

- **Size sorted portfolio's** split the stock universe in 10 portfolio's sorted by market capitalisation.
- **Book-to-market portfolio's** split the stock universe in 10 portfolios sorted by the following ratio: book value of equity / market value of equity, or BE / ME.

Then, these are split into 5 quintiles according to seize and then each quintile is split into 5 BM quintiles, which makes for a total of 25 portfolio's.

Size 1					Size 2					Size 3					Size 4					Size 5				
1				5	1				5	1				5	1				5	1				5

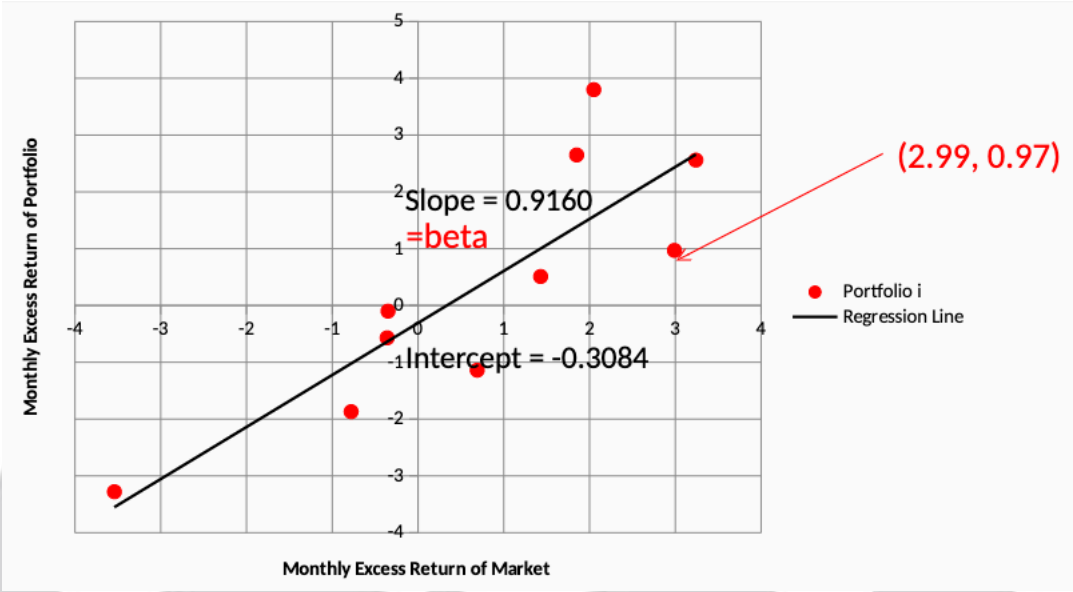
For portfolio *I*, run time series regression.

Date	$r_{i,t}$	$r_{M,t}$	$r_f = 0.4\%$	$r_{i,t} - r_f$	$r_{M,t} - r_f$
Jan	1.37	3.39		0.97	2.99
Feb	-0.17	0.04		-0.57	-0.36
Mar	0.91	1.83		0.51	1.43
Apr	-0.74	1.09		-1.14	0.69
May	-2.88	-3.14	⇒	-3.28	-3.54
Jun	0.30	0.05		-0.10	-0.35
Jul	-1.47	-0.38		-1.87	-0.78
Aug	4.20	2.45		3.80	2.05
Sep	3.05	2.25		2.65	1.85
Oct	2.96	3.64		2.56	3.24

Mean: 0.353%

Mean: 0.722%

Then, the correlation between portfolio *i* and the market portfolio can be illustrated with these data points in the following graph. this is the **first pass regression**.



This asset gives us a mean excess return of 0.353 and a β-estimate of 0.916. This means the collection point is (0.916, 0.353). If this is repeated for several portfolio's or assets, it can be observed that a higher slope (β) results in a higher expected return. All such points lie (approximately) on a straight line and can be checked with the **second pass regression**.

The example that is shown above already shows **promising** results because the intercept is not significantly different from zero and the Pr(>|t|) is greater than 10% (or 5%, or 1%), that is, zero is within the range of the estimation error. The intercept corresponds to the unexplained expected return.

The CAPM predicts a (perfect) **positive linear relationship** between monthly excess return of the portfolio and the β. However, Fama found evidence that suggests that this relationship is false.

Algorithmic and High Frequency Trading

Asset returns can be calculated by using computers, to automate the trading. The idea is to automate trades on certain rules. The **superior computing power** can be used to generate excess returns.

The world is moving towards a world in which only computers do the trades, and even the making of choices for people. There could be a system in which non-humans are involved. One could wonder where the information then comes from.

Very early algorithmic traders were **market makers**. They were paid by the stock exchange to provide liquidity, and thus were obliged to offer bid and ask prices. Market makers guarantee that any investor will get a price to sell, as well as a price to buy. One can always buy or sell, it just depends on the price. The **spread** is the difference between the price for which a market maker would buy and sell for. In an efficient market, the spread is tiny.

One can only profit from **arbitrage** if he is the first one to take the opportunity at it. There is high profitability possible of the **high frequency trading industry**. If it is possible to trade on the millisecond, data structures will be different to analyse. The long-term becomes less important.

Algorithmic trading is about executing orders using automated **pre-programmed instructions**. It leverages speed and computational resources of computers. It aims at a different market **microstructure** which has increased complexity. Computers can also trade on information.

High Frequency Trading (HFT) is a special case of algorithmic trading. But here are various strategies to algorithmic trading;

Systemic trading is a methodical, faster way of classic trading strategies. For example, passive index trading.

Trading ahead of index fund rebalancing targets index funds that need to rebalance portfolios to adjust to new prices and market capitalisation periodically. The timing, volume and direction of these trades is known. Algorithmic traders can exploit this by trading against the rebalancing needs.

Pairs trading concerns trading close substitutes long or short to exploit mispricing in the market. It is risky however. The idea is to identify stocks that are close to each other. Meaning they correlate in structure (one goes up, other goes down for example). If one stock breaks out, then you want to trade to profit from deviation, hoping that they will synch up again. It is based on the belief that history will carry over.

Greek-neutral trading is about the idea that you can calculate the change in value of portfolio given that some other parameter changes. This other parameter is usually the under lying stock value. The “greeks” are parameters of sensitivity of a portfolio with respect t an underlying asset. So delta-neutral mens that the portfolio value remains the same, regardless of what the underlying does. It needs to be constantly adjusted over time since other parameters change. A delta neutral portfolio would mean that the value of the portfolio does not change if the stock price of some stock price changes by a marginal unit.

(Pure) arbitrage is when identical asset trades at different prices on different markets, this is a violation of law of one price. Two assets with identical (future)cash flows trade at different prices. Assets with a known certain future cash flow trades at a different price than the discounted future value.

Mean reversion is about identifying the trading range of a stock and compute the correct average price under certain assumptions. The asset is bought and sold as the price deviates from the correct average price.

Trend chasing is about defining the corridor in which a price should be moving (similar to mean reversion). The asset is bought and sold if the price breaks out of this corridor, assuming it will continue to move in that direction. If a stock breaks out, it is probably because of something fundamental. We know from research that momentum is a thing, so if a fundamental change occurs, it keeps on going. So you define a corridor. One anticipates on a further continuation of a trend.

Scalping is strategy that involves opening and closing positions in very short time periods to profit from bid-ask spread. It is similar to traditional market makers. Some principles of scalping are; take spread as bonus, low exposure and low risk, small price movements, small capital commitment per trade with a high number of trades. It is not allowed by all brokers since it puts pressure on the brokerage system. The idea is not to manipulate the market, but to exploit what is in the market. It is

market making. They looked at the spread and took positions on both sides of the spread. They then provided liquidity. Scalping takes this to the extreme. It is looking at the spread and in milliseconds look at the orders. It is extreme liquidity provision. So normal investors cannot provide bids anymore, they are not able to provide liquidity as normal investors anymore.

Dark pools are about Alternative Trading Systems (ATS). They are used to hide trades and intentions from public exchanges, so there is no order book. They are available to institutional investors, wealthy individuals and HFT'ers. There is anonymous trading with a partner and avoids potential price adjustments based on large order. The trades are only reported after execution. Dark pools. Another market outside the regular market. All it means, one trades peer to peer in an own 'market". They are essentially alternative trading systems. It is anonymously collecting orders. If you don't want the attention before selling or buying large amounts, dark pools are the way to go. There are no records, except when a trade goes through.

Sniffer algorithms make money on other algorithms. There are a lot of algorithms in the market. And if you know there is an algorithm that trades on trend chasing, there is nothing easier than to exploit it if it is there, because it can be predicted. Sniffer algorithms are in market to detect which algorithms are active in the market. The idea is to detect other systematic trading. It is to detect the strategy and systematically exploit it. It can predict trend-chasing an make money on the other algorithm.

There are crashes on the market that look like big crises, but the crashes last very short. It can be a glitch in an algorithm. There are several of these **flash crashes** out there. They are everywhere. They predominantly occur in currency markets because they see a number of up to 93% of the volume traded by algorithms. There are a lot of arbitrage opportunities here.

Illegal strategies are strategies that manipulate prices in any way without the intent of trading. This manipulation is a little bit fuzzy. Sometimes it can look like someone is driving up prices and manipulating the market but can be out of good intentions. So there are always debates between the regulators and traders. It is not like this manipulation is unique to high frequency traders. It is trying to drive up prices and make the market think the high price is right and then selling, or the other way around if you are trying to buy of course.

For **quote stuffing**, it is difficult to definitely say someone has been engaging in this strategy. You place an order, without having the intention to execute it. So you cancel the order before it is executed. If you do this a lot, the bank of the stock exchange cannot handle all these orders. The quotes are stuffed, so programmes are slower. So, no one can trade against it. By that, you drive up the prices without the risk of ever executing the order. For example, if a hedge fund wants to sell some shares, it places a greater number of orders. It is then canceled, other trading programmes observe this and react. They expect the price to go up, they put in own purchase orders, your own orders are canceled. You can then trade against them. It is about small percentage points, but there is a lot of money to be made. The question is how one can prove this.

The idea of **front-running** is very simple. It is, you are abusing your position as a broker for example. You have a client and act on behalf of them on a market. If they place a large order, you go ahead and place a small order first. You trade on anticipation of a large order of a client. You get a large purchase order and before placing this order, you buy some shares for yourself. The larger purchase order is executed, then the price goes up. You can sell your own shares to your client for a higher price. It is executing a small order on your own account.

With **spoofing**, one tries to create an illusion of buying and selling pressure. One puts pressure on the order book, hoping that others react. You place one order on one side of the order book, at the

same time many orders with high volume on the other side. You make sure they do not get executed and cancel them in time. Others will look at this order book and see that there is some pressure, and they react to this. Once these orders come in, you execute the order you wanted to execute initially. And once that happens you get out of the market completely.

Socially Responsible Investing

The **main goals** of a business firm are;
- Maximise shareholder value, so maximising profits
- Creating shared value, so not just profit
- Maximise shareholder wealth, so more long termed and consistent with interests of other stakeholders and society.

Socially responsible finance;
- **Corporate Social Responsibility** (CSR) is a code that you as an investor look for in companies. If they act socially responsible.
- **Socially Responsible Investing** (SRI) is just the act of investing socially responsible.

Everything that we want to do in terms of sustainability has been done by the UN. There are some form of commonly accepted goals that we strive to achieve. Whether we actually get close is a discussion itself.

CSR is about taking the society at large into account. Shareholder **wealth** should be maximised. the idea behind it is that corporations will **self regulate** on this. They will realise that the firm can only be successful if they act socially responsible. Therefore, there is no need for outside regulation. They feel responsible from themselves. The idea behind implementing is on the top level. Everything the company does should adhere to the **standards** of; human rights, labor standards, environmental standards, legal compliance.

This is broken down into the following three domains; environmental, social and corporate governance (ESG). In theory, you can formulate models that would result in no need of external intervention. Because if there is a preference within society, it will automatically translate to the **preference function** of any business and adhere to the generally accepted standards. It can be observed that this idea is emerging.

In practice, it is somewhat criticised to leave it to self regulation. This is because if you have corporate responsibility, one can call it anything they like. It is a matter of communicating. It becomes a **competition of communication**: window-dressing or green-washing.

There is a different way of looking at it. In the end, regulators are there to find those external effects and **regulate** where markets cannot function. If we argue that firms do it themselves, we transfer the role to the firms. Those that have money will direct what CSR is. This should however be governed by the people. For anything that politicians should do, there should be external regulators. We now see more laws with sustainable standards.

If we look at it from an economics perspective, there are clear **arguments** why CSR should be **profitable** for firms. Imagine a world with two companies, with one being socially responsible and the other not, the company that is will probably have higher costs, but also more benefits if investors are willing to provide better funding. Also, they will be more **attractive** to customers and employees. The empirical picture is more or less unclear. Looking at all companies, those with socially responsible activities have higher value. The issue is that the richer companies have more media attention and therefore are more in need of socially responsible activities. They can also

easier finance the social responsibility. In conclusion, the link between CSR and firm value if unclear.

There are theoretical arguments for why CSR can have a positive effect;

- **Employees**. If you are an employee that is somewhat conscious about the environment, you would rather work for a company that is also sensitive towards sustainability. When you work there, your productivity and moral will be up. You there is higher attraction for recruitment for companies that engage in CSR. Companies with high CSR also provide better support for their workforce and therefore have higher retention rates. Recruitment can also be done for a lower cost. These findings can be seen in the empirical evidence.
- **Customers**. They will vote with their wallet and will buy products that they like from companies that they like. If a company engages in CSR activity generates attention. There is little evidence. If people are asked to pay money for a more expensive CSR product, they rather not do it. So the price customers want to pay for a CSR product is very small. There is no big benefit when it comes to customers. If you are the only one that is not, it gets picked up by the customers. People are more observant to losses than to gains, and thus to negative information. So negative outliers have an effect and these firms will be avoided.
- **Regulators**. From theory, regulation is costly. If firms are free to do what you like, they will optimise. Regulation disturbs this environment. So any regulation should be a cost and thus not beneficial. However, it is observed that regulation can put pressure on firms to rethink themselves which in the end is beneficial. Thus, regulation is not necessarily costly.
- **Resource efficiency**. In the end, making a process more efficient, is using the resources that we have. So, any CSR activity that is connected to the idea of sustainability, is directly related to the cost. what we see is that managers heavily underestimate the benefits from being more efficient in resource use and the costs associated with this. That is why there is no intrinsic motivation for being sustainable.
- **Cost of capital**. The idea of how it costs to get money to then invest. The cheaper the cost of capital, the more projects a firm can realise. The idea behind CSR affecting a firms cost of capital is an information asymmetry argument. When financing is provided, there is some form of risk involved. The manager is better informed than the investor. So the more one knows about a company, the lower the associated costs that one has because of information asymmetry. An investor rather invests in a company that he knows than in a company that he does not know. CSR generates a lot of attention. So there is a direct link through attention. So, CSR reduces perceived information asymmetry. Also, ethical investors avoid sin-stocks, which increases the cost of capital.

SRI is something that regards the perspective of the investors. SRI is trying to invest differently than what investors usually do. They do it with more conscious, **focussing on ESG criteria**. The idea is to have a long term investment. It is still investing, but to generate profit but also take long term **societal impact** into account. It is more about the whole shareholder value. This way of investing has grown a lot recently.

The theory of SRI is quite simple. A traditional investor focuses on risk and return. A SRI investor has something else in their **utility function**. There is risk, return and something related to whatever that **investor beliefs**. These can be religious ethical, or political. This reduces the problem to a simple number again, by measuring preferences.

Looking at it from a market perspective is also simple. The EMH says the **prices are right**. If the prices are right, then we should not systematically be able to have any extra return in any way sort or form. The extra return that one would see for SRI would already be **included in the price**. No matter if we are socially responsible, the market price does not care, the price will be efficient. If an

SRI investor excludes any form of investment because of some preference, they are just taking out one of the stocks. And thereby, they will not be able to be better off. Thus, there should not be a systemic benefit from investing socially responsible.

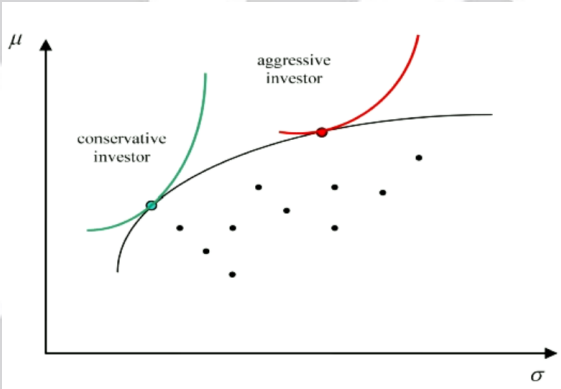
Despite individual stocks or industries having systematically positive returns, so lower prices, the strategies earn positive abnormal returns. So SRI generates a higher return, which is weird. There are two widely used SRI strategies.

- Overweight firms with **high ESG** score, which is **positive screening**.
- **Exclude sin industries**, which is **negative screening**.

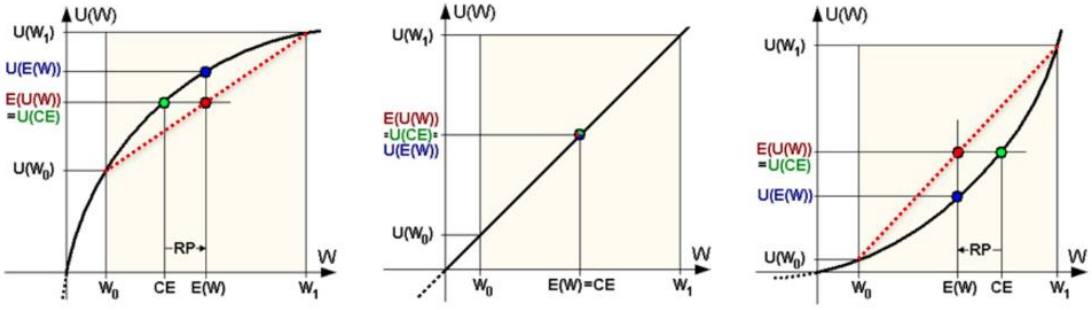
Mutual fund managers make the important decisions. There is increased demand from private investors and currently. There are **legislative mandates** to include ESG factors for public funds. Within the mutual fund industry, the SRI mutual funds do not outperform other mutual funds, they are about equal. So the message is that one can do good for free.

Risk Preferences and Perception

These lines in the graph below are **indifference curves** for investors for different portfolios with different volatility and return. You can see the efficient frontier. The line on the graph is the efficient frontier. Which points one will choose will depend on his risk preferences. One can be a conservative or aggressive investor.



These graphs show **utility functions** of people with different risk preferences. We are interested in how the expected utility of the given portfolio looks compared to the expected value of the portfolio.



The **certainty equivalent** makes the individual indifferent between the amount ant the gamble. if you are offered the certainty equivalent or the gamble, you are indifferent. If you get more money than the certainty equivalent, you take the money, and vice versa. This certainty equivalent has to be equal to the expected utility of the gamble. If the certainty equivalent is lower than the expected value of the gamble, it means that you want to avoid risk and thus are risk averse.

The difference between the expected value of the gamble and the certainty equivalent is the **risk premium**. This number is positive for a risk averse person and negative for a risk seeking person. It is the amount of money that one wants to get in order to be compensated for the risk.

The person on the left is **risk averse**. The expected utility of the portfolio is lower than the utility of the expected value of the portfolio. If this is true, then there are risk averse preferences. The certainty equivalent of the portfolio has to be lower than the expected value of the portfolio.

The person in the middle is **risk neutral**. This is just a linear function. The certainty equivalent is exactly the same as the expected value of the gamble. The risk premium is zero.

The person on the risk is **risk seeking**. For this person the certainty equivalent is higher than the expected value of the gamble. This means that the risk premium is negative. They love risk so much that they are willing to take the gamble in stead of a certain amount of money.

- Risk averse preferences: $CE < E(W) \Rightarrow RP > 0$
  example: $U(W) = \sqrt{(W)}$
- Risk neutral preferences: $CE = E(W) \Rightarrow RP = 0$
  example: $U(W) = 2W$
- Risk seeking preferences: $CE > E(W) \Rightarrow RP < 0$
  example: $U(W) = W^2$

Assume that we have a prospect (P): 35% chance of winning €200, and 65% chance of winning €250. Your utility function is $U(W) = \ln(W)$. Calculate the certainty equivalent of this prospect. What type of risk preferences do we face here?

First calculate the expected utility of the prospect:

$$EU(P) = 0.35\ln(200) + 0.65\ln(250) \approx 5.44$$

Then calculate CE by using that $U(CE) = EU(P) \Rightarrow$

$$CE = \exp(EU(P)) = e^{5.44} \approx 230.44(231.22)$$

How to decide on the risk preferences if we don't know the function? $\rightarrow$ Calculate $E(P)$ and compare it with CE.

$$E(P) = 0.35 \cdot 200 + 0.65 \cdot 250 = 232.5$$

As $CE < E(P)$, we have risk averse preferences.

Most people do not like risk, and are therefore risk averse. This is also found in the literature. Most of the people behave according to risk averse preferences. Because of that, there are several measures of risk aversion. Two measures are considered here.

Arrow-Pratt measures of risk aversion

- *Absolute risk aversion:* $AR(x) = -\frac{u''(x)}{u'(x)}$
- *Relative risk aversion:* $RR(x) = -\frac{xu''(x)}{u'(x)}$

In both cases it is about wealth and how risk taking behaviour changes if wealth changes. For **absolute risk aversion**, considered is the amount of money invested. And if someone has increasing absolute risk aversion, that means that if his wealth increases, he wants to have fewer amount invested in the risky asset. If someone's absolute risk aversion is increasing it means he wants to hold less in the risky asset when his wealth increases. That is why the invested amount decreases. If someone has constant absolute risk aversion, it means that as his wealth increases, the invested amount will not change. Decreasing absolute risk version is also about the amount you hold in risky assets. If someone's wealth increases, he has lower risk aversion. This means he want to invest more money.

For **relative risk aversion**, the fractional wealth invested is considered. If someone has increasing relative risk aversion it means that the more money he has, the less risk he wants to take. So if wealth increases, he wants to hold a smaller portion of your wealth in risky assets. If someone has constant relative risk aversion, someone always wants to invest the same fraction of your money in risky assets. Decreasing relative risk aversion means that someone wants to invest a higher fraction of his wealth when it increases.

The simplest way to find out people's risk aversion is to ask them. It is clear and easy to understand. But this is of course very subjective. It is not very reliable and besides can not be incentivised. Luckily, there are other measures for risk preferences.

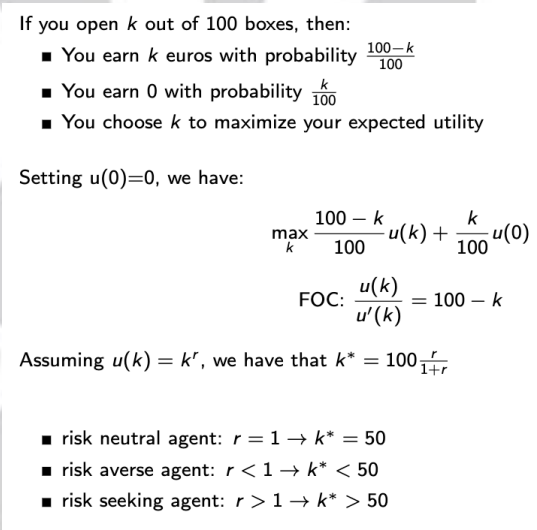
The first method is that subjects can be **asked for the certainty equivalent**. It is about the amount of money that would make him indifferent between the amount and the lottery.

The second method is to **elicit the certainty equivalent stepwise in a dynamic environment**. People are offered an amount or a gamble, if they choose the amount, lower it, if they do not, increase it. The certainty equivalent will eventually be found.

The third one is **presenting a list of lottery pairs**. The subjects are asked between two lotteries. Where they shift, will determine their risk preferences.

People are usually not only risk averse, but also loss averse. We are loss averse if we prefer to avoid losses. A lottery with a 50/50 chance of winning and losing $100 would be rejected. It is even very likely to be rejected when the gain is twice as high as the loss.

A way to elicit risk preferences is the **Bomb Risk Elicitation Task** (BRET). There are 100 boxes and in one is a bomb. Subjects have to decide how many boxes (k) they will open. If the box someone checks has no bomb, they keep going. For every open box, subjects can earn money. If they find the bomb, they lose everything. This is how risk aversion can be measured.



Another way to elicit risk preferences is the **Balloon Analogue Risk Task** (BART). It is kind of similar to the BRET test. You ask respondents to pump a balloon, with each pump they get more money, but also the possibility that it pops increases. If that happens, they get nothing. Subjects get the choice to pump more or stop and cash out. The difference is that one does not know the probabilities, whereas the exact probabilities are known for BRET. Even though it is used to elicit risk preferences, one could also argue that there is some ambiguity since the probabilities are not known. For the calculation it is assumed that the subjects know the maximum pumps and that they know the probabilities.

Assuming that agents know the total number of pumps, K, at step n (in total K pumps), and the probability of explosion, the investor faces

- Winning $0.25(n+1)$ with probability $\frac{K-n-1}{K}$, and 0 with $\frac{n+1}{K}$.
- Winning $0.25n$ for sure

Or equivalently, A: Winning an additional 0.25 with probability $(K-n-1)/K$ or lose $0.25n$ otherwise; B: Nothing.

With n increasing:

- The additional payment stays constant,
- The amount of loss increase,
- The loss probability increases

The agent stops if

$$u(0.25n) > \frac{K-n-1}{K}u(0.25*(n+1)) + \frac{n+1}{K}u(0)$$

Dohmen et al. (2011) not only assessed risk preferences per domain, but also looked at **individual characteristics**. So which characteristics explain more or less risk taking. They found that on the financial domains;

- Females are more risk averse than males
- Older people are more risk verse than younger people
- Taller people are willing to take more risks than shorter people
- People with a better educated father are willing to take more risks

Falk et al. (2018) looked at global preferences about risk attitudes, trust, positive and negative reciprocity, altruism, and time preference. They found that there might be **cultural differences** in risk-taking. For example, higher risk taking in the US than in Europe.

Risk perception is about how likely people believe something is to happen. It is the opinions that people express when they are asked to evaluate hazardous activities, substances and technologies. There might be situations in which people perceive risk differently. This is important because people act based on perception. And while **risk preferences** are assumed to be stable, **risk perception** might change based on the circumstances, people act based on risk perception.

If it is understand how risk perception is influenced, it can be used for policymakers. One can **improve communication** between experts and the public. Also, educational efforts can be directed to help the public better understand risks.

This is all important because portfolio decisions should be made on risk and return, not on risk perception. Literature show that investors look at perceived return and perceived risk.

Factors of perceived risk;

- **Under- or overestimation of risk magnitude**. People tend to overestimate the value of their experience and capabilities and underestimate the associated risk
- **Familiarity**. Working in a familiar situation encourages people to take more risk.
- **Severity of consequence**. When fear of injury or penalty seems low, people are more willing to take risks. People acceptance of risk is based on the seriousness of potential outcome and on how severely they might be injured.
- **Voluntary exposure**. People who voluntary take risk, such as speeding on a highway or working without a hard hat, perceive their action as less dangerous.

- **Personal experience**. Personal stories - preferably told by the person involved - of accidents and dangers create attention and increases risk perception in an audience.
- **Understood hazards**. Hazards that can be clearly explained cause less alarm than those that are not understood and are views as uncontrollable.
- **Cost of compliance**. If the cost of noncompliance is very low, people are less likely to perceive an action as a risk. As an example, more people will risk getting a $20 speeding ticket than a $200 ticket. Cost associated with compliance can be a strong motivator to change risk perception
- **Social influence**. Employees can become role models, benefitting themselves and others, or can be negative influences.

Financial products can be characterised with different properties; mean, standard deviation, loss probability, skewness, kurtosis, semi-variance, expected loss, max. return, min. Return. Zeisberger (2020) looked at how these different factors influenced risk perception. He found that basically the **probability of loss** is the main driver of risk perception. The higher the loss probability, the more risky subjects assessed that given investment and the less they wanted to invest in it. Probability of loss is nearly a perfect driver of risks.

The **experience-description gap** describes that consistent with evolutionary perspective, understanding the different probabilities can depend on how you are presented with these probabilities. People behave differently when they experience something compared to the case when they are just given a description.

Learning in Markets

The **rationality assumption** starts with the idea that individual decision making is based on optimisation. It regards the idea of homo economicus, making decisions based on all information available. Individuals want to maximise utility and companies try to maximise profits. To optimise, agents need to form beliefs. Rational expectations lead to the **optimal solution**. Based on beliefs, individuals take an action or form a strategy. The individual decisions result in aggregate behaviour in the market. If this is a solution of the model, the system is in **equilibrium**. Then, by deviating, an individual is always worse off. In the equilibrium, beliefs are consistent with actions, and actions are consistent with beliefs. In a system with an equilibrium, there is no feedback. Therefore, there is no learning.

There is a second type of rationality, this is **procedural or bounded rationality**. Here, the assumption of the homo economics is relaxed. Agents are rational when they construct the solutions to complex problems, they do not directly solve the problems rationally. However, they do use simple rules, heuristics and rules of thumb. Learning means that the agents are constantly looking for new and better strategies. They do not want to optimise, but want to find **satisfying strategies**. With this, agents can save resources, time and effort when making decisions. So it might be rational to not be rational.

The **as-if defence** states that people should not care about learning because even if people are not rational, at some point they behave as if they were rational and are capable to find the rational way. People can **roughly learn** to behave rationally by trial and error process. Furthermore, this idea argues that markets can end up in an equilibrium because the agents learn to behave as if they were rational. This is the case because you are driven out of the market if you are not rational, like with bank runs. However, this is not necessarily true.

There are two main categories to model learning; individual learning and social learning. They differ in how learning takes place. How do I learn about the optimal action? Does it only depend on my own experience? Or can I also learn from others?

In **individual learning** our agent will look at the perceived set of alternatives and consequences. It is possible that our agents does not look at all alternatives, but at a select few. The agents observe their outcome and update their action the next time based on the performance. They learn about the own set of alternatives and consequences, and they also learn about the behaviour of the opponent, but no their consequences and strategy. To conclude, agents hold certain beliefs about their economic environment and adapt these beliefs, and the corresponding actions as new information based on own experience comes along.

When talking about **social learning**, more agents are taken into account. Again, there is a perceived set of alternatives and consequences. Other agents and their alternatives and consequences can be seen. The agent can now take the most preferred action based on this information. An agent can update their actions based on the outcomes of other agents. In contrast to individual learning, agents hold certain beliefs about their economic environment, share these beliefs, and update their beliefs and corresponding actions when observing others' actions and new available information, which is own and others' experience.

A simple example for **learning as a monopolist** would look like the solution below. There are no costs associated with production and full rationality is assumed. Normal characteristics apply. Also, the SOC tells us that we are sure to find maximum and not minimum.

$$q^M = \underset{q}{argmax}\{P(q)q\} = \underset{q}{argmax}\{(a - bq)q\}$$

FOC:

$$\frac{\partial \pi(q)}{\partial q} = \frac{\partial P(q)q}{\partial q} = a - 2bq = 0 \Leftrightarrow q^* = \frac{a}{2b}$$

SOC:

$$\frac{\partial^2 \pi(q)}{\partial q^2} < 0 \Leftrightarrow -2b < 0$$

However, it is not realistic that the monopolist perfectly knows the demand function and its parameters. So it is about how the monopolist can learn the optimal production level. A more realistic assumption is that only **qualitative information** is available about the inverse demand function. The monopolist knows that if the price rises, the demand decreases, and the other way around.

The first method to learn the optimal quantity is **gradient learning**. It is assumed that the monopolist makes decisions in every period. He chooses how much to produce, and after this production, he can observe the profit and the associated marginal profit, which is characterised by the following model.

$$\pi(q_t) \text{ and } \frac{\partial \pi(q_t)}{\partial q_t}$$

The monopolist **adjust his output in the direction** in which profits increase. If he sees that the profits increase, he will increase the production level even more. But if the profit decreases, they will decrease the production to get closer to the optimal production.

$$q_{t+1} - q_t > 0 \text{ if } \frac{\partial \pi(q_t)}{\partial q_t} > 0$$

$$q_{t+1} - q_t < 0 \text{ if } \frac{\partial \pi(q_t)}{\partial q_t} < 0$$

This leads to the adjusted dynamic under gradient learning. The only thing that the monopolist needs is the marginal profit and he needs to come up with an **adjustment factor**, λ. A smaller λ is more precise, but also slower to find the optimal solution. A big λ could result in jumping around the optimal solution.

$$q_{t+1} = q_t + \lambda \frac{\partial \pi(q_t)}{\partial q_t} \Rightarrow q_{t+1} = q_t + \lambda(a - 2bq_t)$$

About this adjustment process, it is known that the **steady state** of the gradient learning dynamics satisfies the FOC. We are in the steady state if the production in the next period is exactly the same of the previous period. Then, applying the adjustment process will not change the production level. If λ is not too big you will always end up in the maximum if one is in the neighbourhood of the steady state.

For **econometric learning**, the same monopolist is considered. He has a linear demand function or at least thinks it is linear. He tries to estimate the demand function of the following form from available information. Our monopolists looks at past data and has estimated the demand function based on past data. They use the estimated demand function to calculate the profit maximising quantity. He does maximisation and use an estimated demand function based on **past data**. the realised price will likely differ from the estimated price. They then look at new information and can update parameters and reestimate.

$$\tilde{P}(q) = a - bq$$

$$\tilde{P}(q) = a_{t-1} - b_{t-1}q$$

It is assumed that the monopolist observes the realised price. He can also see the marginal inverse demand associated with the production. They see how much the price changes. They take these two and reestimate the linear demand curve.

$$\tilde{P}_t(q) = a_t - b_t q = P(q_t) + P'(q_t)(q - q_t)$$

They want a linear function for which it is true that the realised price based on that estimated function is indeed the price that they can observe. The realised marginal demand based on their estimation is indeed the marginal demand that they can observe. The next production is characterised by the following.

$$q_{t+1} = \frac{a_t}{2b_t} = -\frac{P(q_t) - q_t P'(t)}{2P'(q_t)} = \frac{1}{2}q_t - \frac{P(q_t)}{2P'(q_t)}$$

The **steady state** again satisfies the FOC. The FOC is the first derivative of the profit function. furthermore, local minimal are always unstable.

$$q_{t+1} = \frac{1}{2}q_t - \frac{P(q_t)}{2P'(q_t)}$$

What was demonstrated above is a **simple** example. The monopolist has no cost function and no competitors. In reality there are many different learning models. Also, firms might be limited to different production technologies and switching between them might be costly. Even though you can learn a little bit by moving towards the optimal output, it might not be feasible because it requires a lot of costs. Besides, firms might learn from other producers or markets. The problem might be more complex because it the problem is not constant for example. Lastly, firms need to make inter-temporal decisions and maximise an inter-temporal objective function, the trade-off between experimenting to learn more and exploiting known strategies to make a good enough profit.

If you look at any markets, **expectations** play a big role in market outcomes. Agents form their price forecasts, and based on these forecasts then buy or sell assets. These actions will determine the price in the market. Expectations have a feedback on the realised outcome. Because of this feedback loop, it is also important how people form expectations. A way to look at this is to conduct a so called learning-to-forecast experiment which focuses only on expectation formation.

There is a **law of motion for the price-mechanism**. It is important to know that there is a fundamental price of an asset. The price depends on price expectation in the future. This is because people base their actions of tomorrows actions. If everyone expects the fundamental price, the market ends up in this equilibrium price.

$$p_t = p^f + \frac{1}{1.05}(\bar{p}^e_{t+1} - p^f) + \varepsilon_t$$

Subjects only know **qualitative information**, like how the market works. They know that there is a positive feedback, but not the exact law of motion. After each round, they are informed about the last realised price, but not about others' forecast.

It was found that participants are **unable to learn** the fundamental price. Only sometimes the price moved towards the fundamental price. Even though there are fluctuations and bubbles, participants are **able to coordinate** their forecasting activity. This is also why the bubbles are big, deviations from the fundamental value amplify each other. There is a lot of individual data from this and what has been found is that there is no homogenous expectations model that fits these experiments. This means you can not say that each individual uses one simple heuristics. A heuristic switching model better fits the data.

Genetic algorithm is a social learning model based on evolutionary theories. Agents do not need to have high cognitive abilities, so there is no need for maximisation but they have a set of strategies. They then evaluate the strategies after. The two main principles are;

- **Exploitation** / **selection**: duplicate the most successful strategies for the next period and discard the less successful ones. This is survival of the fittest.
- **Exploration** / **diversity**: introduce new strategies for the next period to be able to find potentially better strategies. So diversity to achieve better economic performance.

At each time period there are one or more populations of chromosomes. Each chromosome codes the decision of the agents. There is initially a random set of strategies. Strategies are updated in every period by reproduction, crossover, mutation, and election. For each period on strategy is drawn from the set of strategy to be used.

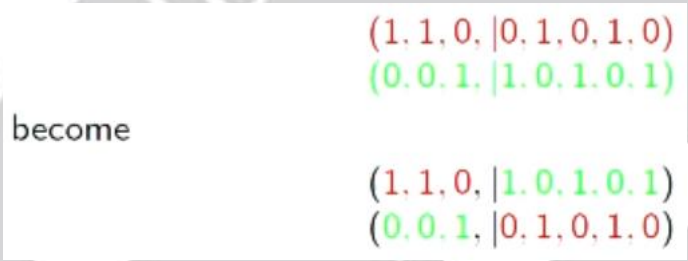
With a probability P_{imit} each period the new set is constructed with one of the two following methods;

- **Tournament selection** randomly draws a subset *n* of the *N* strategies and replicates the best strategies from these *n*.
- **Roulette-wheel selection** replicates a strategy with a probability, which is proportional to its relative fitness $F_{i,t}$ in the population. The higher their performance, the higher the likeliness that they are chosen for the next set.

$$\frac{F_{i,t}}{\sum_{i=1}^{N} F_{i,t}}$$

The chosen strategies go into the mating pool.

With a certain probability, crossover operates on the mating pool. Two strategies are randomly selected from the mating pool. These are the parent strategies. These produce some offspring with the following method. There is a random selection of two strategies and the place where to perform cross-over.



This is how one can experiment with the strategy set. The idea of the **cross-over set** is that agents communicate with each other. They exchange information and by doing so they might alter the strategies that they might use.

With some other probability, **mutation** can happen. This means that you take a strategy from the mating pool and randomly change of one of the 0's to 1 or one of the 1's to a 0. The idea is that you experiment with a very small change to stay in the neighbourhood.

Once we have cross-over and mutation. We still need to look at whether our strategies with cross-over and mutation are good enough. Because if they are worse than the parent strategy, it does not make sense to introduce them. **Election** tests whether these newly generated strategies can enter the strategy set at all. It calculates the potential fitness of these new strategies, and compares it to the actual fitness of the parents. If the fitness is higher than at least one of the parents, it replaces one of the parent strategies.

We need to choose a strategy to actually bring to the market. **Selection** selects which one of the strategies should be played in the subsequent period, this happens with tournament or roulette-wheel selection.

Learning-to-forecast experiments are not the only type of trade experiments, experiments directly measuring trading behaviour in the lab are more popular. The usual design of is based on Smith, Suchanek & Williams (SSW, 1988) in which subjects have cash and asset. They can trade on a double-auction. The asset pays stochastic dividend and trading is for a known, usually 15 periods.

Disclaimer

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