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-Project Financial Instruments-

Chapter 1: Introduction

- OTC market (Over the counter) → exchange your futures and options
  - prior crisis free from regulation, now rules are set for participants
  - Forward contracts are the counterpart of OCT and future contracts

Futures contracts

- Futures contract → an agreement to buy or sell an asset at a certain time in the future for a certain price.
  - e.g. in June a trader wants to buy 5000 corn for the September delivery, this will be communicated to the broker. Then a trader wants to sell 5000 corn in the September delivery, and this will too be communicated to the broker. Now a price can be determined and the deal would be done.
  - The buyer has agreed to buy a *long futures position*
  - The sellers has agreed to sell a *short futures position*
  - the price is known as the *futures price* → it is determined by laws of supply and demand
  - If both parties agree upon a price, they both have a binding contract.
  - *futures price* is often not equal to the *spot price* (price for -almost- immediate delivery)
  - Often standardized contracts
  - Traded on an exchange and are standardized contracts
- Future contracts are made to reduce risks of scarcity and oversupply for both seller and buyer.

The over the counter market

- OCT markets are smaller
- participants contact each other directly or use an interdealer broker
- Banks often act as market makers, they make a *bid price (*at which they are prepared to take one side of the derivatives transaction) and an *offer price* (at which they are prepared to take the other side.
- *Bid price*: The price that a dealer is prepared to pay for an asset
- *Offer price*: the price that a dealer is offering to sell an asset
- After crisis more regulation → more rules to improve transparency of OCT markets, improve market efficiency and reduce systemic risks.
- Sometimes OCT is forced to become more like exchange-traded market:
  - Standardized OCT derivatives should be traded on *swap execution market* (platform where participants can post bid and offer quotes and accept them)

- There is a *central clearing party* for standardized derivatives transactions. Their role is to stand between the two sides in an over-the-counter derivative transaction.
    - All trades must be reported to a central registry
- Over the counter market is much larger than exchange traded markets

Forward Contracts

- Forward contract: is similar to a futures contracts in that it is an agreement to buy or sell an asset at a certain time in the future for a certain price. But forward contracts are traded in the OCT market.
- Forward contracts on foreign exchange are very popular.
- Most banks employ both *spot and forward foreign exchange traders.*
- *Spot traders* are trading a foreign currency for almost immediate delivery
- *Forward traders* are trading for delivery at a future time.
- *Bid* (what the bank is prepared to buy GBP for)
- *Offer* (what the bank is prepared to sell GBP for)
- The *bid* and *offer* quotes are for very large transactions.
- Are traded in the over-the-counter market.

Options

- Options are traded both on exchanges and in the over-the-counter markets.
- Two types of options
    - Call option, gives the holder the right to buy an asset by a certain date for a certain price.
    - Put option, gives the holder the right to sell an asset by a certain date for a certain price.
- *Exercise / Strike price* = price in the contract
- *Expiration date / maturity date* = the date in the contract
- *European option* can only be exercised only on the maturity date
- *American option* can be exercised at any time during its life.
- An option gives the holder the right to do something, the holder does not have to exercise this right. This is different from futures (or forward) contracts.
- So an option gives the holder a right to do something but does not oblige him to do so, while a future (forward) contract forces the holder to buy / sell his assets.
- Therefore, entering a futures contract costs nothing, while an option costs an *option premium* which has to be paid upfront.
- The price of a call option decreases as the strike price increases. (the higher the price you want to buy it for, the less premium you have to pay)
- The price of a put option increases as the strike price increases. (the higher the price you want to sell it for, the more premium you have to pay)
- Both types of options tend to become more valuable as their time to maturity increases.
- e.g an investor buys one December call option contract on Google with a strike price of $580 the offer price is $35.30. Therefore $3530 is remitted to the other party. The

investor has obtained at a cost of $3530 the right to buy 100 Google shares for $580 each. If the price of Google does not rise above $580.00 by December, the option is not exercised and the investor loses $3530. But if Google does well and the stock is $650 then the investor is able to buy 100 shares for $580 and sell them immediately for $650 for a profit of $7000, or $3470 when the initial costs of the options are taken into account.

- An alternative. Selling one September put option contract with a strike price of $540 at a bid price of $19.80. This would lead to an immediate cash inflow of $1980. If Google stock prices stays above $540, this option is not exercised and the investor makes a $1980 profit. However, if the stock price falls and the option is exercised when the stock price is $500 there is a loss. The investor must buy 100 shares at $540 while they are $500 worth. This leads to a loss of $4000, or $2020 taken the $1980 profit into consideration.
- There are 4 types of participants
    - Buyer of calls
    - Seller of calls
    - Buyer of puts
    - Seller of puts
- Buyers are referred to as having *long positions.* Sellers are referred as having *short positions.*
- *Writing the option* = selling an option

Types of trader

- Three broad categories of trader can be identified
    - Hedgers → use futures, forwards and option to reduce the risk that they face from potential future movements in a market variable.
    - Speculators → use them to bet on a future direction of a market variable
    - Arbitrageurs → take offsetting positions in two or more instruments to lock in a profit.

Hedgers

- How hedgers can reduce the risks with forward contracts and options
- Hedging with Forward Contracts
    - Example: In June, ImportCo must pay £10 million on September, for goods purchased in Britain. It buys £10 million in the three-month forward market at an offer exchange rate of 1.5585 for the pounds it will pay.
      ExportCo will receive £30 million on September from a customer in Britain. It sells £30 million in the three-month forward market at a bid exchange rate of 1.5579 for the pounds it receive. (Bid prices if you are selling currency to the market. Offer prices if you are buying currency from the market).
    - This will reduce the risk for sudden movements in the exchange currencies.

        - Hedging reduces the risk, but it not necessarily the case that the outcome with hedging will be better than the outcome without hedging.
- Hedging using Options
    - Example: An investor who owns 1000 shares of a company and wants protection against a possible decline in the share price over the next two months. Market quotes are as follows:
        - current share price: $28
        - $27.50 put price: $1.
    - The investor buys option contracts for a total of $1000. This gives the investor the right to sell 1000 shares for $27.50 per share during the next period.
    - Thus if share price drops below $27.50, the investor is still able to sell them for the fixed price of $27.50 and is thus protected
- Fundamental difference between forward contracts and options for hedging. Forward contracts are designed to neutralize risks, while options will provide an insurance. Unlike forwards, options involve the payment of an up-front fee.

Speculators

- Speculators using Futures
    - Example one speculator thinks the british pound will strengthen and wants to speculate on that.

Table 1.4 Speculation using spot and futures contracts. One futures contract is on £62,500. Initial margin for four futures contracts = $20,000

	Possible Trade	
	Buy £250,000 *Spot price = 1.5470*	*Buy 4 futures contracts* *Futures price = 1.5410*
Investment	$386,750	$20,000
Profit if April spot = 1.6000	$13,250	$14,750
Profit if April spot = 1.5000	−$11,750	−$10,250

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    - Difference between spot price and future price: using the futures contracts will only need a small amount of cash ($20000). This will give the speculator a large leverage.
- Speculators using Options.
    - Suppose a speculator considers that a stock is likely to increase in value. He is able to speculate that in two ways, using shares or using options.

Table 1.5 Comparison of profits from two alternative strategies for using $2,000 to speculate on a stock worth $20 in October

Investor's strategy	December stock price	
	$15	$27
Buy 100 shares	−$500	$700
Buy 2,000 call options	−$2,000	$7,000

- using shares → profit = 100 x ($27 - $20) = $700
- using call options → profit = 2000 x $4.50 = $9000 → $9000-$2000=$7000
- Options like futures provides a form of leverage. Good outcomes become very good, while bad outcomes result in the whole initial investment lost.
- Difference between futures and options: When a speculator uses futures, the potential loss as well as the potential gain is very large. When options are used, no matter how bad things get, the speculator's loss is limited due to the amount paid for options.

Arbitrageurs

- Arbitrage involves locking in a riskless profit by simultaneously entering into transatction in two or more markets.

Value of £1: $1.5500

A trader does the following:

- Arbitrage opportunities such as in the example cannot last for long.
    - Laws of supply and demand will set the two prices equivalent quickly.

Dangers

- Derivatives are very versatile instruments
- To avoid problems it is very important for both financial and nonfinancial corporations to set up controls to ensure that derivatives are being used for their intended purpose.
- Risk limits should be set and the activities of traders should be monitored.
- However, mistakes can happen

Chapter 2: Mechanics of Futures Markets.

Opening and closing futures positions

- A contract is usually referred to by its delivery month.

- During that whole month the delivery can be done.
- The party with the *short position* chooses when delivery is made.
- Most investors choose to close out their positions prior to the delivery period specified in the contract. → often expensive to make or take delivery.
- Closing a position involves entering into an opposite trade to the original one that opened the position.
    - e.g an investor buys five July corn futures contracts on May 6, can close out the position on June 20 by selling (i.e. shorting) five July corn future contracts. e.g. An investor who sells (i.e. shorts) five July contracts on May 6 can close out the position on June 20 by buying five July contracts.
- In each case the investor's total gain or loss is determined by the change in the futures price between May 6 and June 20

Specification of a futures contract

- A new contract should specify in some detail the exact nature of the agreement between the two parties.
    - The asset
    - The contract size (how much will be delivered under one contract)
    - Where will the delivery be made
    - When will the delivery be made
- The party with the short position (the party that sells the asset) chooses what will happen when alternatives are specified by the exchange.
- *Notice of intention to deliver* → the party with the short position is ready to deliver.
- The asset
    - When the asset is a commodity (marketable item) there can be differences in quality
    - therefore the exchange stipulates the grades of the commodity that are acceptable.
    - Some commodities have a range of grades (each grade comes with its own price)
    - Financial assets in futures contracts are generally well defined and unambiguous.
- The contract size
    - Specifies the amount of the asset that has to be delivered under one contract
        - (if too large it will be unable to be used, if too small it is too expensive due to associated trading costs)
    - The correct size for a contract clearly depends on the likely user
- Delivery arrangements
    - The place where the delivery will be made must be specified by the exchange
    - When alternatives are specified, the short position party may choose.
- Delivery months
    - A futures contract is referred by its delivery month

        - Contract must specify the exact period when delivery can be made
        - Delivery months vary, chosen by the exchange to meet the needs of participants
        - Trading generally ceases a few days before the last day on which delivery can be made
- Price quotes
    - The exchange defines how prices are quoted ($1.00 or $1.000)
- Price limits and position limits
    - Daily price movement limits are specified by the exchange
    - *A limit move* is a move in either direction equal to the daily price limit.
    - trading ceases for one day when limit is met.
    - Purpose is to prevent large price movements from occurring due to speculative excesses.
    - *Position limits* are the maximum number of contracts that a speculator may hold.
    - Purpose is to prevent speculators from exercising undue influences on the market.

Accounting and Tax

- Accounting standards require changes in the market value of a futures contract to be recognized when they occur unless the contract qualifies as a hedge.
- If hedge, then gains or losses are recognized for accounting purposes in the same period in which the gains or losses from the item being hedged are recognized → *Hedge accounting*
- Example: consider a company with a December year end.

Example 2.1 Accounting treatment of a futures transaction

A company buys 5,000 bushels of March 2014 corn in September 2013 for 750 cents per bushel and closes out the position in February 2014 for 780 cents per bushel. The price of March 2014 corn on December 31, 2013, the company's year end, is 770 cents per bushel.

If contract is not a hedge, the treatment of these transactions leads to:
Accounting profit in 2013 = 5,000 × 20 cents = $1,000.
Accounting profit in 2014 = 5,000 × 10 cents = $500.

If contract is hedging a purchase of corn in 2014, the result is:
Accounting profit in 2013 = $0.
Accounting profit in 2014 = 5,000 × 30 cents = $1,500.

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- Tax
    - Two key issues are the nature of a taxable gain or loss and the timing of the recognition of the gain or loss.
        - Gains or losses are classified as capital gains/losses or as part of ordinary income

- For corporate taxpayer, capital gains are taxed at the same rate as ordinary income and the ability to deduct losses is restricted.
    - Capital losses are deductible only to the extent of capital gains
- For a noncorporate taxpayer, short term capital are taxed at the same rate as ordinary income, but long-term capitals taxed at a lower rate. (long term > 1 year)
- Positions in futures contracts are treated as if they are closed out on the last day of the tax year.
- The tax regulations define a hedging transactions as a transaction entered into the normal course of business, because of these reasons:
    - To reduce the risk of price changes or currency fluctuations with respect to property that is held or to be held by the taxpayer for the purposes of producing ordinary income
    - To reduce the risk of price or interest rate changes or currency fluctuations with respect to borrowings made by the taxpayer.
- Gains or losses from hedging transactions are treated as ordinary income.

Chapter 4: Interest Rates.

Types of rates

- An interest rate in a particular situation defines the amount of money a borrower promises to pay the lender.
- The interest rate applicable in a situation depends on the credit risk, the higher the credit risk the higher the interest rate that is promised by the borrower.
- *Treasury Rates*
    - Treasury rates are the rates an investor earns on Treasury bills and Treasury bonds.
    - used by government to borrow in its own currency
    - it is assumed that there is no change the government will default on an obligation denominated in its own currency
    - often assumed to be totally risk-free
- *LIBOR*
    - London Interbank Offered Rate.
    - It is a reference interest rate, produced once a day, designed to reflect the rate of interest rate at which banks can obtain unsecured loans from other banks.
    - Banks has to be creditworthy to be able to borrow at LIBOR.
    - Not always risk free for the banks.
- *Repo rates*
    - Repurchase agreement, Repo.
    - This is a contract where an investment dealer who owns securities agrees to sell them to another company now and buy them back later at a slightly higher price.
    - So the other company borrows money to the investment dealer. The interest rate for the company is called the *repo rate*.

- *overnight repo* → agreement is renegotiated each day
- *a term repo* → long-term arrangements.

Measuring interest rates

- Annual compounding = $X * (interest rate)^t
- Compounded rate per annum → A $\left(1+\frac{R}{m}\right)^{mn}$
  - R = interest rate
  - *m* = compounded m times per year
  - *n* = amount of years
- Continuous compounding → Ae^{Rn}
- Discounting at continuous compounding → Ae^{-Rn}
- Suppose R_c = rate of interest with continuous compounding
- and R_m = rate of compounding *m* times per annum.

$$Ae^{R_c n} = A\left(1+\frac{R_m}{m}\right)^{mn}$$

$$e^{R_c} = \left(1+\frac{R_m}{m}\right)^{m}$$

$$R_c = m \ln\left(1+\frac{R_m}{m}\right)$$

$$R_m = m(e^{R_c/m} - 1)$$

- then :

Theories of the term structure of interest rates

- what determines the shaper of the zero curve
  - *Expectations theory*: long-term interest rates should reflect expected future short-term interest rates → forward interest rate corresponds to a certain future period is equal to the expected future zero rate interest for that period.

- *Market segmentation theory* → no relationship between short-, medium-, and long-term interest rates. → determined by supply and demand in each market
- *Liquidity preference theory* → investors prefer to preserve their liquidity and invest funds for a short periods of time. Borrowers prefer to borrow at fixed rates for long periods of time.

- *Liquidity preference theory*
  - *Net interest income* → excess of the interest received over the interest paid and needs to be carefully managed.
  - Long term rates tend to be higher than those that would be predicted by expected future short-term rates.
  - Banks will ensure that interest rates on deposits and mortgage are so that the net interest income > 0. This means that maturities for deposits are higher for long-term rates than short-term rates. And that mortgages rates are lower for short-term and higher for long-term.
  - The asset/liability management group has to ensure that the maturities of assets on which interest is earned and the maturities of the liabilities on which interest paid are matched.
- Liquidity
  - A portfolio where maturities are mismatched can lead to liquidity problems

Chapter 5: Determination of Forward and Futures Prices

Investment Assets vs. Consumption assets

- Forward and future contracts, distinguish between *investment assets* and *consumption assets*.
  - *Investment asset*: an asset that is solely held for investment purposes by significant numbers of investors (e.g stocks/bonds/gold/silver).
  - *Consumption asset*: an asset primarily held for consumption, e.g. copper, oil, pork
- we can use arbitrage arguments to determine the forward and futures prices of an investment asset from its spot price and other observable market variable.

Short selling

- One strategy is short selling
- *Short selling*: selling an asset that is not owned. This is possible for some, but not all investment assets.
- suppose an investor instructs a broker to short 500 shares of company X. The broker will carry out the instructions by borrowing the shares from someone who owns them and selling them in the market in the usual way. At some later stage, the investor will close

out the position by purchasing 500 shares of company X in the market. These are used to replace the shares that were borrowed, so that the short position is closed out. the investor takes a profit if the stock price has declined and a loss if it has risen. If the broker has to return the borrowed shares earlier, the investor is forced to close out the position. The investor has to pay dividends or interest to the one that has lend the securities.

Table 5.1 Cash flows from short sale and purchase of shares

Purchase of shares	
April: Purchase 500 shares for $120	−$60,000
May: Receive dividend	+$500
July: Sell 500 shares for $100 per share	+$50,000
	Net profit = −$9,500
Short sale of shares	
April: Borrow 500 shares and sell them for $120	+$60,000
May: Pay dividend	−$500
July: Buy 500 shares for $100 per share	−$50,000
Replace borrowed shares to close short position	
	Net profit = +$9,500

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- The investor is required to maintain a *margin account* with the broker
- *Margin account* consists of cash (or marketable securities) deposited by the investor with the broker to guarantee the investor will not walk away if the share price rise.
- If prices rise and additional margin is not provided, the short position is closed out.

Assumptions and notation

- The following will be assumed regarding most market participants
  - They are subject to no transactions cost when they trade.
  - They are subject to the same tax rate on all net trading profits
  - They can borrow money at the same risk-free rate of interest as they can lend money.
  - They take advantage of arbitrage opportunities as they occur.
- it is the trading activities of these key market participants and their eagerness to take advantage of arbitrage opportunities as they occur that determine the relationship between forward and spot price.
- *Risk free rate*: The rate at which money is borrowed or lent when there is no credit risk so that the money is certain to be repaid.
- LIBOR can be used as a proxy for *risk free rate*.

Forward price for an investment asset.

Action in 3 months:	*Action in 3 months:*
Sell asset for $43	Buy asset for $39

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- This first arbitrage works when the forward price > $40.50.
- The seconds arbitrage works when the forward price < $40.50.
- So no arbitrage when price = $40.50.

Example 5.1 Forward price of an asset providing no income

Consider a four-month forward contract to buy a zero-coupon bond that will mature one year from today. (This means that the bond will have eight months to go when the forward contract matures.) The current price of the bond is \$930. We assume that the four-month risk-free rate of interest (continuously compounded) is 6% per annum. Because zero-coupon bonds provide no income, we can use equation (5.1), with $T = 4/12$, $r = 0.06$, and $S_0 = 930$. The forward price, F_0, is given by

$$F_0 = 930e^{0.06\times 4/12} = \$948.79$$

This would be the delivery price in a contract negotiated today.

[2] For another way of seeing that equation (5.1) is correct, consider the following strategy: buy one unit of the asset and enter into a short forward contract to sell it for F_0 at time T. This costs S_0 and is certain to lead to a cash inflow of F_0 at time T. So S_0 must equal the present value of F_0; that is, $S_0 = F_0e^{-rT}$, or equivalently $F_0 = S_0e^{rT}$.

- Short sales are not possible for all assets. → however not a problem
- suppose the underlying investment asset gives rise to no storage costs or income
  - If $F_0 > S_0e^{rT}$ an investor can adopt the following strategy
    1. Borrow S_0 dollars at an interest rate *r* for *T* years.
    2. Buy one unit of the asset.
    3. Short a forward contract on one unit of the asset.
  - At time *T* the asset is sold for F_0. an amount of S_0e^{rT} is required to repay the loan at this time and the investor makes a profit of $F_0 - S_0e^{rT}$
  - If $F_0 > S_0e^{rT}$ an investor who owns the asset can:
    1. Sell the asset for S_0.
    2. Invest the the proceeds at interest rate *r* for time *T*.
    3. Take a long position in forward contract on the asset.
  - At time *T*, the cash invested has grown to S_0e^{rT}. The asset is repurchased for F_0 and the investor makes a profit of $S_0e^{rT} - F_0$ relative to the position the investor would have been if the asset had been kept.
  - We can expect the forward price to adjust so that neither of the two arbitrage opportunities we have considered exists. This means that the relationship in equation must hold.

Are forward prices and futures prices equal?

- When there is no uncertainty about future interest rates, the forward price for a contract with a certain delivery date is in theory the same as the future price for a contract with that delivery date.
- When interest rates vary unpredictably, they are not longer the same in theory.
- The prices may be different due to taxes, transaction costs and use of margins.
- It is reasonable to assume that forward and futures prices are the same.

Futures prices of stock indices

- how index futures prices are determined
- A stock index can usually be regarded as the price of an investment asset that pays dividends.
- The investment asset is the portfolio of stocks underlying the index, and the dividends paid by the investment asset are the dividends that would be received by the holder of this portfolio.

Example 5.5 Calculation of index futures price

Consider a three-month futures contract on the S&P 500. Suppose that the stocks underlying the index provide a dividend yield of 1% per annum, that the current value of the index is 1,300, and that the continuously compounded risk-free interest rate is 5% per annum. In this case, $r = 0.05$, $S_0 = 1{,}300$, $T = 0.25$, and $q = 0.01$. Hence, the futures price, F_0, is given by $F_0 = 1{,}300e^{(0.05-0.01)\times 0.25} = \$1{,}313.07$.

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- If $F_0 > S_0e^{(r-q)T}$, profits can be made by buying spots (i.e. for immediate delivery) the stocks underlying the index and shorting futures contracts.
- If $F_0 < S_0e^{(r-q)T}$, profits can be made by shorting or selling the stocks underlying the idnex and taking a long position in futures contracts
- This is called *Index Arbitrage*.

Chapter 7: Swaps

- Swap is an over-the-counter derivatives agreement between two companies to exchange cash flows in the future.
  - It defines the state when the cash flows are to be paid
  - It defines the way in which they are to be calculated
- Whereas a forward contract is equivalent to the exchange of cash flows on just one future date, swaps typically lead to cash-flow exchanges taking place on several future dates.
- common type of swap is a ''plain vanilla'' interest rate swap → . It is the exchange of a fixed rate loan to a floating rate loan
- Floating rate is often LIBOR rate (rate that banks are prepared to lend money to other banks with AA credit rating)
- *Notional principal*: The principal is only used for the calculation, but itself is not changed
- Swaps could be used to transform a floating-rate loan into a fixed-rate loan.
  - e.g Microsoft has to pay LIBOR +10 basis points → LIBOR + 0.1%
  - Due to swap it gets LIBOR back
  - It pays 5% under the terms of the swap
  - → pays net interest rate of 5.1% thus a fixed loan
- It can also be the other way around

Example 7.1 The versatility of swaps

Swaps have been so successful because they can be used in so many different ways. The swap in Figure 7.1 can be used to transform the nature of liabilities or assets.

Transforming Liabilities

Figure 7.2 shows that the swap can be used by Microsoft to switch its borrowings from floating to fixed and by Intel to do the reverse:

	Microsoft	*Intel*
Loan payment	LIBOR + 0.1%	5.2%
Add: Paid under swap	5.0%	LIBOR
Less: Received under swap	−LIBOR	−5.0%
Net payment	5.1%	LIBOR + 0.2%

Transforming Assets

Using the same swap, Figure 7.3 shows Microsoft can switch its assets from fixed to floating and Intel can do the reverse:

	Microsoft	*Intel*
Investment income	4.7%	LIBOR − 0.2%
Less: Paid under swap	−5.0%	−LIBOR
Add: Received under swap	LIBOR	5.0%
Net income	LIBOR − 0.3%	4.8%

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- Swaps can also be used to transform the nature of an asset
  - e.g earning at a fixed rate to earning at a floating rate of interest
  - suppose Microsoft own 100 million in bonds with 4.7% interest / year
    - It receives 4.7% on the bonds
    - It receives LIBOR under the terms of the swap
    - It pays 5% under the terms of the swap
    - → now receives LABOR -0.3% interest.
- Financial intermediary institutions often receive fees of 3-4 basis points (0.03-0.04%)
- Often financial institutions act as market makers and start taking swaps without a counterparting company → it has to hedge risks really carefully

Day count issues

- day count conventions affect payments on a swap

Confirmations

- *Confirmation* is the legal agreement underlying a swap and is signed by representatives of the two parties

The comparative-advantage argument

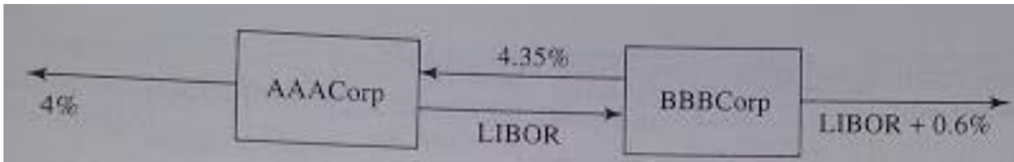
- Some companies have a comparative advantage when borrowing in fixed-rate markets, whereas others have a comparative advantage in floating-rate markets.
- Thus they go to the market where to have a comparative advantage
- Swaps are used to transform fixed into floating and vise versa
- 

Figure 7.6 Swap agreement between AAACorp and BBBCorp when the rates in Table 7.4 apply
    - AAAcorp agrees to pay BBBcorp interest at six-month LIBOR on $10 million. In return BBBcorp pays AAAcorp 4.35% interest rate on $10 million per year
    - For AAAcorp this means:
        - It pays 4% per year to outside lenders
        - It receives 4.35% per year from BBBcorp
        - It pays LIBOR to BBBcorp
        - → thus AAAcorp pays LIBOR - 0.35%, this is 0.25% less than it would pay if it went to the floating markets direcly
    - For BBBcorp this means:
        - it pays LIBOR + 0.6% per year to outside lenders
        - it receives LIBOR from AAAcorp
        - It pays 4.35% per year to AAAcorp
        - → thus BBBcorp pays 4.95% per year, that is 0.25% less than it would pay if it went to the fixed rating market directly
- Why are the spread between rates be different in fixed and floating markets
    - Due to the nature of contracts available to companies in fixed and floating market
    - Fixed rate → negotiated over 5 years
    - Float rate → negotiated LIBOR every 6 months
    - It depends on the creditworthiness and probability to default from companies
    - In 6 months the chance it will default is smaller than in 5 years → thus bigger change between companies with different creditworthiness in 5 year rates

Nature of swap rates

- A financial institution can earn the five year swap rate by:
    - lending the principal for the first six months to a AA-borrower and then relending it for successive 6 months… and then again..
    - Entering into a swap to exchange the LIBOR income for the 5-years swap rate
- Thus 5-year swap rate is equal to 10 consecutive times 6-month LIBOR rate

Overnight indexed swaps

- Overnight-index swaps are related to the unsecured overnight borrowing and lending that banks do at the end of each day to satisfy their liquidity needs
- *Overnight indexed swap* (OIS) = a swap where a fixed rate for a period (e.g 1-month or 3-months) is exchanged for the geometric average of the overnight rates during the period. (daily compounded interests)
- Often short lived contracts (3 months or less)
- e.g. a bank (bank A) can engage in the following transactions
    - Borrow $100 million in the overnight market for three months, rolling the interest and principal on the loan forward each night
    - Lend the $100 million for three months at LIBOR to another bank (bank B)
    - Use an OIS to exchange the overnight borrowings for borrowings at the three months OIS rate (constant rate)
    - → result bank A receives three month LIBOR rate and paying the three months OIS rate (risk premium). → often OIS rate is lower than the LIBOR rate so small profit for bank A.

Valuation of interest rate swaps

- 1. Calculate forward rates for each of the LIBOR rates that will determine swap cash flows
- 2. Calculate the swap flows on the assumption that LIBOR rates will equal forward rates
- 3. Discount the swap cash flows at the risk-free rate.
- *Forward rate agreement* (FRA) = each exchange of payment in an interest rate swap is a FRA where interest at a predetermined fixed rate is exchanged for interest at LIBOR floating rate.

Estimating the zero curve for discounting

- Derivative practitioners have traditionally used LIBOR and swap rates to define a risk-free zero curve and have used this curve to calculate discount rates when valuing derivatives.
- when LIBOR rates and swap rates are used to define discount rates, the value of a newly issued floating-rate bond that pays LIBOR is always equal to its principal value
    - → This due to that the bond provides a rate of interest of LIBOR and LIBOR is the discount rate so that the interest on the bond exactly matches the discount rate.
- for a newly issued swap, the value of the fixed-rate bond equals the value of the floating-rate bond.
- the value of the floating-rate bond equals the notional principal.

Example 7.3 Determining zero rates from swaps

Suppose that the 6-month, 12-month, and 18-month LIBOR/swap zero rates have been determined as 4%, 4.5%, and 4.8% with continuous compounding and that the 2-year swap rate (for a swap where payments are made semiannually) is 5%. This 5% swap rate means that a bond with a principal of $100 and a semiannual coupon of 5% per annum sells for par. It follows that, if R is the 2-year zero rate, then

$$2.5e^{-0.04\times0.5} + 2.5e^{-0.045\times1.0} + 2.5e^{-0.048\times1.5} + 102.5e^{-2R} = 100$$

Solving this, we obtain $R = 4.953\%$. (Note that this calculation is simplified in that it does not take the swap's day count conventions and holiday calendars into account. See Section 7.2.)

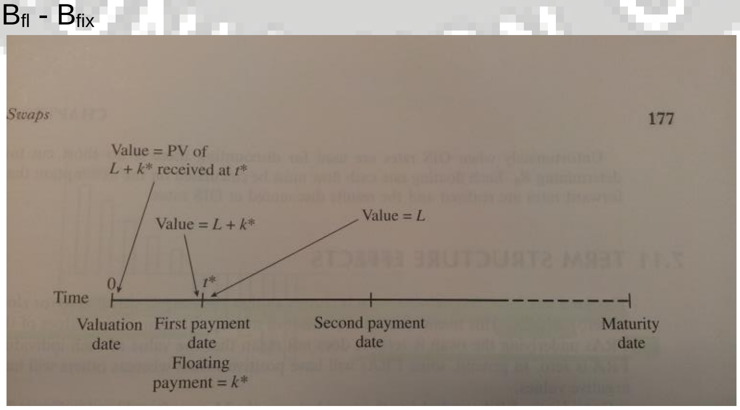
- If OIS rates are used for risk-free rates for discounting, similar principle as LIBOR/Swap rates.
- The one-month OIS rate defines the one-month zero rate, the three month defines the three month… etc

Forward rates

- The calculation of forward LIBOR rates depends on the rates used for discounting.

Valuation in terms of bonds

- Principal payments are not exchanged in an interest rate swap
- However, if we assume they are paid → a swap where fixed flows are received and floating cash flows are paid can be regarded as long position in a fixed-rate bond and a short position in a floating -rate bond.
    - $V_{swap} = B_{fix} - B_{fl}$
    - V_{swap} = value of the swap
    - B_{fix} =value of the fixed rate payments underlying the swap
    - B_{fl} = is the value of the floating rate bond underlying the swap
- If floating cash flows are received and fixed cash flows are paid then →
    - $V_{swap} = B_{fl} - B_{fix}$



- Discounting will result in the value of the floating-rate bond today is $(L+k^*)e^{-r^*t^*}$
  - L = notional principal
  - k* = floating payment made at time t*
  - t* = when the payment will be made
  - r* = LIBOR/swap zero rate for a maturity of t*

Term structure effects

- The fixed rate in an interest swap is chosen so that the swap is worth zero initially.
  - This means that the outset of a swap the sum of values of the FRA's underlying the swap is zero.
- Term structure of interest rate is upward sloping at the time the swap is negotiated → this means that the forward interest rates increase as the maturity of the FRA increases.
- Term structure of interest rate is downward sloping at the time the swap is negotiated → this means that the forward interest rates decrease as the maturity of the FRA increase.

Fixed-for-fixed currency swaps

- *fixed-for-fixed currency swap*: This involves exchanging principal and payments at a fixed rate of interest in one currency for principal and payments at a fixed rate of interest in another currency.
- fixed-for-fixed currency swap → the interest rate in each currency is at a fixed rate.
  - e.g. at the outset of the swap IBM pays $15 million and receives £10 million. Each year during the life of the swap contract IBM receives $0.90 million (6% of the $15 million) and pays £0.50 million (5% of the £10 million).
- A fixed-for-fixed currency swap can be used to transform borrowings in one currency to borrowings in another currency.
- The initial exchange of principal converts the proceeds of the bond issue from U.S dollars to sterling.
- The swap can also be used to transform the nature of assets
- Currency swaps can be motivated by comparative advantage.

Valuation of Fixed-for-fixed currency swaps

- Like interest rate swaps, fixed-for-fixed currency swaps can be decomposed into either the difference between two bonds or a portfolio of forward contracts
- If we define V_{swap} as the value in US dollars of an outstanding swap where dollars are received and a foreign currency is paid then
  - $V_{swap} = B_D - S_0B_F$
  - B_F = value of the bond in the foreign currency by foreign cash flows on the swap

- B_D = value of the bond defined by the domestic cash flows on the swap
- S_0 = is the spot exchange (expressed as number of dollars per unit of foreign currency)

- Each exchange of payments in a fixed-for-fixed currency swap is a forward foreign exchange contract.
- The value of the currency swap is normally close to zero initially and immediately after the initial exchange of the principal.
- However, when interest rates in two currencies are significantly different then
    - the payer of the highest-interest currency has negative values in the early exchange of cash flows and positive towards the end
    - the payer of the low-interest currency is in the opposite position

Other currency swaps

- Fixed-for-floating: Where a floating interest rate in one currency is exchanged for a fixed interest rate in another currency
- Floating-for-floating: Where a floating interest rate in one currency is exchanged for a floating interest rate in another currency.

Credit risk

- Contracts such as swaps that are private arrangements between two companies entail credit risks
- There is a chance that one party will get into financial difficulties and default. The financial institution then still has to honor the contract it has with the other party
- In practice it is likely that the counterparty would choose to sell the contract to a third party or rearrange its affairs in some way so that its positive value in the contract is not lost.
- In swaps, it is sometimes the case that the early exchanges of cash flows have positive values and the later exchanges have negative values.
- Potential losses from defaults on a swap are much less than the potential losses from defaults on a loan with the same principal. → due to the value of the swap is usually only a small fraction of the value of the loan
- Potential losses from defaults on a currency swap are greater than on an interest rate swap.
- It is important to distinguish between the credit risk and market risk to a financial institution in any contract.
    - credit risk : chance of default by the counterparty
    - market risk: change of market variables such as interest rates and exchange rates
- To reduce credit risk in over-the-counter markets, regulators require standardized over-the-counter derivatives to be cleared through central clearing parties (CCPs)
    - CCPs acts as an intermediary between the two sides in a transaction.

- *Credit default swap:* is like an insurance contract that pays off if a particular company or country defaults

Summary

- two most common types of swap
  - interest rate swaps
  - currency rate swaps
- interest rate swap: one party agrees to pay the other party interest at a fixed rate on a notional principal for a number of years. In return, it receives interest at a floating rate on the same notional principal for the same period of time.
- Currency rate swap: one party agrees to pay interest on a principal amount in one currency. In return, it receives interest on a principal amount in another currency
- Principal amounts are not usually exchanged in an interest rate swap.
- An interest rate swap can be used to transform a floating rate loan into a fixed-rate loan or vice versa.
- A currency swap can be used to transform a loan in one currency to another currency
- There are two ways of valuing interest rate and currency swaps
  - swap is decomposed into a long position in one bond and a short position in another bond
  - it is regarded as a portfolio of forward contracts.
- Financial institutions entered in swaps are exposed to credit risks.

Chapter 9: Mechanics of Option markets

9.1 Types of options

- *Call option*: gives the holder of the option the right to buy an asset by a certain date for a certain price.
- *Put option:* gives the holder the right to sell an asset by a certain date for a certain price.
- *Expiration date* and the *maturity date* are stated on the contracts.
- *American option*: can be exercised at any time up to the expiration date
- *European option*: can only be exercised at the expiration date itself.
- *Call options*
  - Option will only be exercised when the stock price at expiration date is > the price of the call option
  - Profit is then → (current price of the stock - call price) * number of stocks - costs of the options.
  - However if the difference in price of stock and call price is small, there could be still a potential loss
  - In general: call options should always be exercised at the expiration date if the stock price is above the strike price.

Example 9.1 Profit from call option

An investor buys a call option to purchase 100 shares.

Strike price = \$100
Current stock price = \$98
Price of an option to buy one share = \$5
The initial investment is 100 × \$5 = \$500

At the expiration of the option the stock price is \$115. At this time, the option is exercised for a gain of

$$(\$115 - \$100) \times 100 = \$1{,}500$$

When the initial cost of the option is taken into account, the net gain is

$$\$1{,}500 - \$500 = \$1{,}000$$

-

- *Put options*
    - *the purchaser of a put option is hoping that the price of a stock will decrease.*
    - If the stock price is lower at that moment, the investor can buy those stocks and resell them at the put price.

Example 9.2 Profit from put option

An investor buys a put option to sell 100 shares.

Strike price = \$70
Current stock price = \$65
Price of put option to sell one share = \$7

The initial investment is 100 × \$7 = \$700.

At the expiration of the option, the stock price is \$55. At this time, the investor buys 100 shares and, under the terms of the put option, sells them for \$70 per share to realize a gain of \$15 per share, or \$1,500 in total. When the initial cost of the option is taken into account, the net gain is

$$\$1{,}500 - \$700 = \$800$$

-

9.2 Option positions

- There are two sides to every contract.
    - the long position → has bought the option
    - the short position → has sold the or written the option
    - the writer of an option receives cash up front, but has potential liabilities later.
- there are four types of option positions
    - A long position in a call option
    - A long position in a put option
    - A short position in a call option
    - A short position in a put option
- Payoff from a long position in a European call option is
    - max(S_T - K, 0)
        - with K = strike price
        - S_T = final price
        - this reflects that the option will be exercised if $S_T > K$ and not when $S_T < K$

- The payoff to the holder of a short position in the European call option is
  - - max(S_T - K, 0) = min(K - S_T, 0)
- The payoff to the holder of long position in a European put option is
  - max(K - S_T, 0)
- The payoff from a short position in a European put is
  - - max(K - S_T , 0) = min(S_T - K, 0)

9.3 Underlying assets

- Trading of options on stocks, currencies, stock indices, and futures
- Stock options
  - Most trading is on exchanges
  - One contract gives the holder the right to buy or sell 100 shares at the specified strike price.
- Foreign Currency Options
  - Most currency options trading is now in the over-the-counter market, but there is some exchange trading
- Index options
  - Many different option currently trade throughout the world in both the over-the-counter market and the exchange-traded market.
  - One contract is to buy or sell 100 times the index at the specified strike price.
- Future options
  - When an exchange trades a particular futures contract, it often also trades options on that contract
  - A futures option normally matures ust before the delivery period in the futures contract.

9.4 Specification of stock options

- Expiration dates
  - Longer term options, LEAPS
- Strike prices
  - The exchange normally chooses the strike prices at which options can be written so that they are spaced $2.50, $5 or $10 apart.
  - If the stock prices moves outside the range defined by the highest and lowest strike price, trading is usually introduced in an optin with a new strike price.
- Terminology
  - Call options are referred as:
    - *In the money:* if Stock price > Strike price
    - *At the money*: if Stock price = Strike price
    - *Out the money:* if Stock price < Strike price
  - Put options are referred as:
    - *In the money:* if Stock price < Strike price
    - *At the money*: if Stock price = Strike price
    - *Out the money:* if Stock price > Strike price

- *Intrinsic value* of an option is defined as the maximum of zero and the value the option would have if it were exercised immediately.
- Flex options and other nonstandard products
    - *flex options*: traders agree to nonstandard terms
        - (e.g. different expiration date / strike price than usual)
    - Options on exchange traded funds
    - *Weeklys*: options that expire the week after
    - *Binary options*: Provide a fixed payoff if the strike price is reached
    - *Credit event binary options, CEBOs*: similar like credit default swap. Fixed payoff will be given when a credit event occurs.
    - *DOOM options*: also like the credit default swap. Provide a payoff if the price of the underlying asset plunges.
- Dividends and Stock splits
    - The early over-the-counter options were dividend protected.
    - Exchange-traded options are not usually adjusted for cash dividends
    - Exchange-traded options are adjusted for stock splits
    - After an *n*-for-*m* stock split, the strike price is reduced to *m/n* of its previous value, and the number of shares covered by one contract is increased to *n/m* of its previous value.
    - Stock options are adjusted for stock dividends
        - a 20% stock dividend means that investors receive one new share for for each five already owned
        - The stock price can be expected to go down as a result of a stock dividend.
- Position limits and exercise limits
    - *Position limit*: the maximum number of option contracts that an investor can hold on one side of the market
    - *Exercise limit:* the maximum number of contracts that can be exercised by andy individual, in five consecutive business days
    - They are designed to prevent the market from being unduly influenced by the activities of an individual investor or group of investors

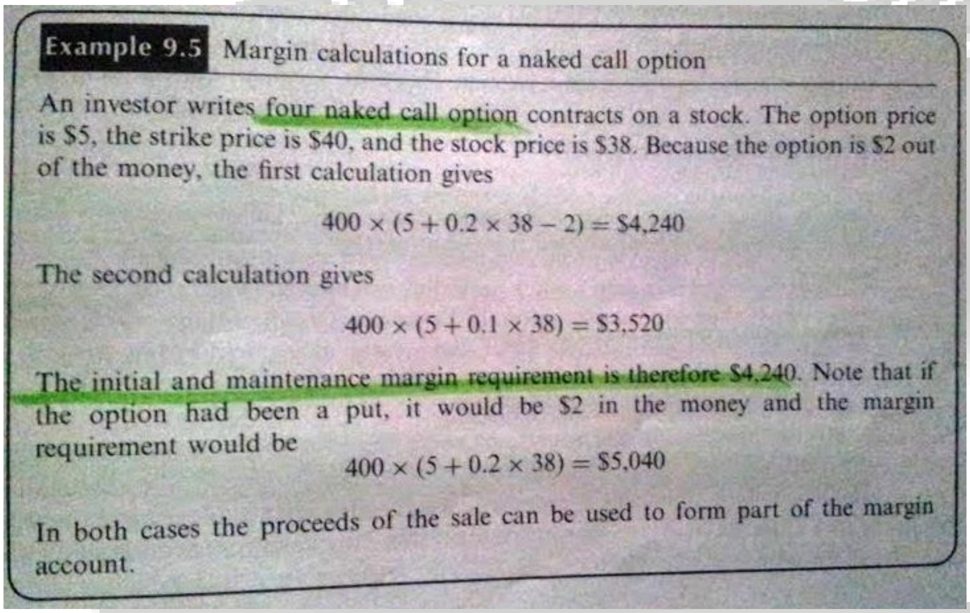
9.5 Trading

- Market makers
    - Most options exchange use market makers
        - he/she will quote both a bid and an offer price on the option
    - Bid is the price the market maker is prepared to buy
    - Offer is the price at which the market maker is prepared to sell
    - The existence of market maker ensures that buy and sell order can always be executed at some market price without any delays.
    - Market makers therefore add liquidity to the market.
    - The market makers themselves make their profit from the bid-offer spread
- Offsetting orders

- An investor who has purchased an option can close out the position by issuing an offsetting order to sell the same option.

9.7 Margin requirements

- *Buying on margin:* pay using borrowed money using a margin account
- Investors are not allowed to buy options on margin because it would create a very high level of leverage rate.
- A trader who writes options is require to maintain funds in a margin account.
- Writing naked options
    - is an option that is not combined with an offsetting position in the underlying stock



Example 9.5 Margin calculations for a naked call option

An investor writes four naked call option contracts on a stock. The option price is \$5, the strike price is \$40, and the stock price is \$38. Because the option is \$2 out of the money, the first calculation gives

$$400 \times (5 + 0.2 \times 38 - 2) = \$4{,}240$$

The second calculation gives

$$400 \times (5 + 0.1 \times 38) = \$3{,}520$$

The initial and maintenance margin requirement is therefore \$4,240. Note that if the option had been a put, it would be \$2 in the money and the margin requirement would be

$$400 \times (5 + 0.2 \times 38) = \$5{,}040$$

In both cases the proceeds of the sale can be used to form part of the margin account.

9.8 The option clearing corporation

- the Option Clearing Corporation (OCC): it guarantees that options writers will fulfill their obligations under the terms of options contracts and keeps a record of all long and short positions.

9.9 Regulation

- Options markets are regulated
    - OCC → has rules of governing the behaviour of trades
    - Option markets have a willingness to regulate themselves

9.11 Warrants, employee stock options and convertibles

- *Warrants*: are options issued by a financial institution or nonfinancial corporation
- *Employee stock options*: call options issued to employees by their company to motivate them to act i the best interest of the company's shareholders.
- *Convertible bonds*: are bonds issued by a company that can be converted into equity at certain times using a predetermined exchange ratio

Summary

- Call: give the holder the right to buy the underlying asset for a certain price by a certain date
- Put: gives the holder the right to sell the underlying asset for a certain price by a certain date.
- An exchange must specify the terms (size, expiration time, strike price)
- Most option exchange use market makers (make quote bid and offer)
- Market makers improve liquidity of the market
- Writers of options have potential liabilities and are required to maintain margin with their brokers.
- Many options are trader in the over-the-counter market → more easy to make tailor made options.

Chapter 10: Properties of Stock Options

10.1 Factors affecting option prices

- There are six factors affecting the price of a stock option
    1. Current stock price, S_0
    2. The strike price, K
    3. The time to expiration, T
    4. The volatility of the stock price, σ
    5. The risk-free interest rate, r
    6. The dividends that are expected to be paid

	call	put	call	put
Current stock price	+	−	+	−
Strike price	−	+	−	+
Time to expiration	?	?	+	+
Volatility	+	+	+	+
Risk-free rate	+	−	+	−

- Stock price and strike price
    - If a call option is exercised at some future time, the payoff will be the amount by which the stock price exceeds the strike price.
        - call options therefore become more valuable as the stock price increases and less valuable as the strike price increases.
    - For a put option the payoff on exercise is the amount by which the strike price exceeds the stock price.
        - put options become more valuable as the stock price decreases and less valuable as the strike price decreases
- Time to expiration
    - Both put and call American options become more valuable (or at least do not decrease in value) as the time to expiration increases

- The long-life option must therefore always be worth at least as much as the short-life option
- European put and call options usually become valuable as the time to expiration increases, this is not always the case.
- the dividend will cause the stock price to decline, so that the short-life option could be worth more than the long-life option.

- Volatility
  - the volatility of a stock price is a measure of how uncertain we are about future stock price movements.
  - As volatility increases, the chance that the stock will do very well or very poorly increases.
  - The values of both calls and puts increases as volatility increases (you are still insured, but can profit majorly from the volatility.
- Risk-Free interest rate
  - As the interest rate in the economy increase, the expected return required by investors from the stock increases.
    - increases the value of call options and decrease the value of put options
  - In practice however, when interest rates rise (fall), stock prices tend to fall (rise).
  - As the interest rate decreases → increase the value of a call option and decrease the value of a put option.
- Dividends
  - Dividends have the effect of reducing the stock price on the ex-dividend rate.

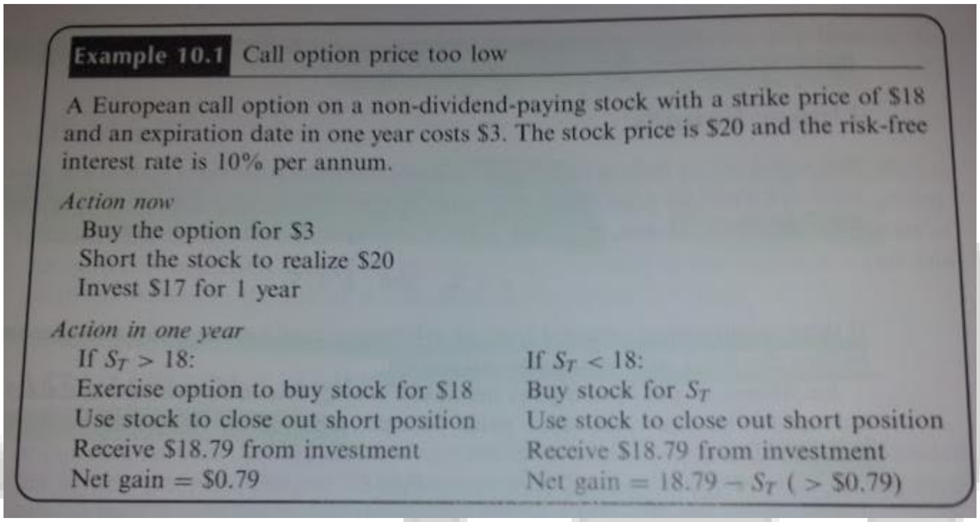
10.2 Notations

- S_0 = Current stock price
- K = Strike price of option
- T = Time to expiration of option
- S_T = Stock price on expiration date
- r = Continuously compounded risk-free rate of interest for investment maturing in time T.
  - this is nominal rate of interest, not the real rate.
- C = Value of American call option to buy one share
- P = Value of American put option to sell one share
- c = Value of European call option to buy one share
- p = Value of European put option to sell one share

10.3 Upper and lower bounds for option prices.

- If an option price is above the upper bound or below the lower bound, there are profitable opportunities for arbitrageurs
- Upper bounds
  - the call option can never be more worth than the stock → stock price is an upper bound to the option price $c \leq S_0$ and $C \leq S_0$

- the put option can never be more worth than the strike price → $P \leq K$ and for European put options → $p \leq Ke^{-rT}$
- Lower bounds
    - For European call option on a non-dividend-paying stock is → $S_0 - Ke^{-rT}$



Example 10.1 Call option price too low

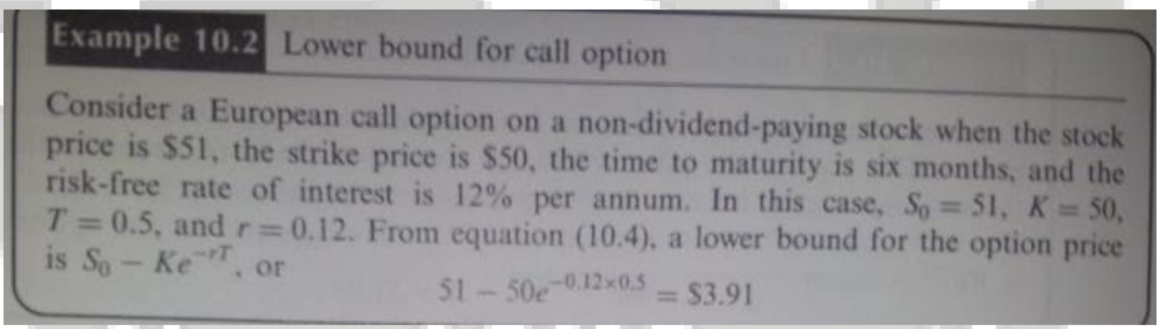
A European call option on a non-dividend-paying stock with a strike price of \$18 and an expiration date in one year costs \$3. The stock price is \$20 and the risk-free interest rate is 10% per annum.

Action now
Buy the option for \$3
Short the stock to realize \$20
Invest \$17 for 1 year

Action in one year

If $S_T > 18$:	If $S_T < 18$:
Exercise option to buy stock for \$18	Buy stock for S_T
Use stock to close out short position	Use stock to close out short position
Receive \$18.79 from investment	Receive \$18.79 from investment
Net gain = \$0.79	Net gain = $18.79 - S_T$ (> \$0.79)

- thus → $c \geqslant S_0 - Ke^{-rT}$ or $c + Ke^{-rT} \geqslant S_0$

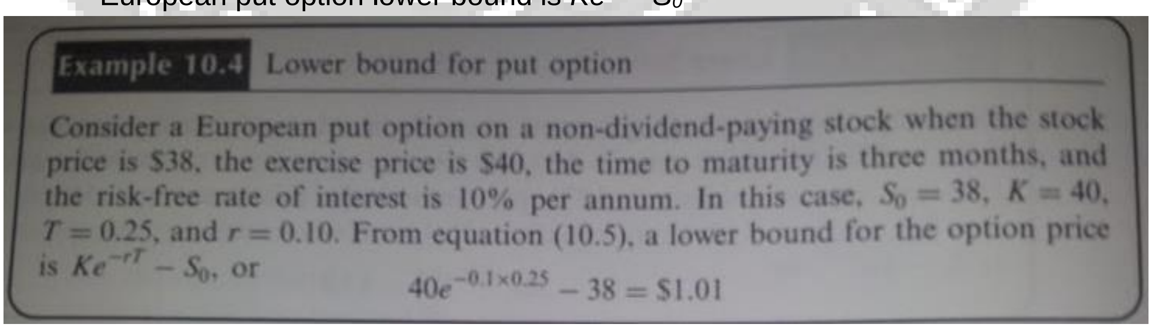


Example 10.2 Lower bound for call option

Consider a European call option on a non-dividend-paying stock when the stock price is \$51, the strike price is \$50, the time to maturity is six months, and the risk-free rate of interest is 12% per annum. In this case, $S_0 = 51$, $K = 50$, $T = 0.5$, and $r = 0.12$. From equation (10.4), a lower bound for the option price is $S_0 - Ke^{-rT}$, or

$$51 - 50e^{-0.12\times0.5} = \$3.91$$

- Lower bounds for puts on non-dividend-paying stocks
    - European put option lower bound is $Ke^{-rT} - S_0$



Example 10.4 Lower bound for put option

Consider a European put option on a non-dividend-paying stock when the stock price is \$38, the exercise price is \$40, the time to maturity is three months, and the risk-free rate of interest is 10% per annum. In this case, $S_0 = 38$, $K = 40$, $T = 0.25$, and $r = 0.10$. From equation (10.5), a lower bound for the option price is $Ke^{-rT} - S_0$, or

$$40e^{-0.1\times0.25} - 38 = \$1.01$$

thus → $p + S_0 \geqslant Ke^{-rT}$ *or* $p \geqslant Ke^{-rT} - S_0$

10.4 Put-Call parity

Table 10.2 Values of Portfolio A and Portfolio C at time T

		$S_T > K$	$S_T < K$
Portfolio A	Call option	$S_T - K$	0
	Zero-coupon bond	K	K
	Total	S_T	K
Portfolio C	Put Option	0	$K - S_T$
	Share	S_T	S_T
	Total	S_T	K

- *Put-Call parity*: it shows that the value of a European call with a certain exercise price and exercise date can be deducted from the value of a European put with the same exercise price and exercise date, and vice versa.
- Put-call parity only holds for European options
- $c + Ke^{-rT} = p + S_0$

10.5 Calls on a non-dividend paying stock

- it is never optimal to exercise an American call option on a non-dividend-paying stock before the expiration date. IF the investors plans to keep the stock for the remaining life of the option
- If the stock is currently overpriced → the investors is better off selling the option than exercising it.
- There are two reasons an American call on a non-dividend paying stock should be exercised early:
    - It insures the holder against the stock price falling below the strike price
    - the time value of money → from the perspective of the option holder, the later the strike price is paid out the better.

10.6 Puts on a non-dividend-paying stock

- It can be optimal to exercise an American put option on a non-dividend-paying stock early
- Like a call option, a put option can be viewed as providing insurance.
- the early exercise of a put option becomes more attractive as S_0 decreases, as r increases and as the volatility decreases.
- provided that $r > 0$, it is always optimal to exercise an American put immediately when the stock price is sufficiently low.
- Because there are some circumstances when it is desirable to exercise an American put option early, it follows that an American put option is always worth more than the corresponding European put option.

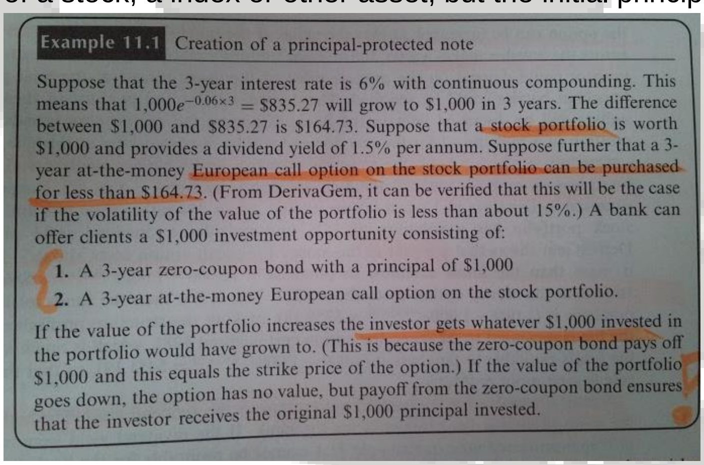
10.7 Effect of dividends

- Lower bounds for Calls and Puts
    - $c \geqslant max(S_0 - D - Ke^{-rT}, 0)$
    - D = present value of the dividends during the life of the option
    - $p \geqslant max(D + Ke^{-rT} - S_0, 0)$
- Early exercise
    - sometimes it is optimal to exercise an American call immediately prior to an ex-dividend date. It is never optimal to exercise a call at other times
- Put-Call parity
    - Put-call parity becomes
        - $c + D + Ke^{-rT} = p + S_0$
        - $S_0 - D - K \leqslant C - P \leqslant S_0 - Ke^{-rT}$

Chapter 11: Trading strategies involving options

11.1 Principal-protected notes

- Investors who are risk averse will take a principal protected notes pattern
- Investors who are willing to take more risk could choose a bull or bear spread
- Investors who are even more willing to take risks would take a straightforward long position in a call or put option
- *Principal-protected notes:* the return earned by the investor depends on the performance of a stock, a index or other asset, but the initial principal amount invested is not at risk.



Example 11.1 Creation of a principal-protected note

Suppose that the 3-year interest rate is 6% with continuous compounding. This means that $1{,}000e^{-0.06\times3} = \$835.27$ will grow to $1,000 in 3 years. The difference between $1,000 and $835.27 is $164.73. Suppose that a stock portfolio is worth $1,000 and provides a dividend yield of 1.5% per annum. Suppose further that a 3-year at-the-money European call option on the stock portfolio can be purchased for less than $164.73. (From DerivaGem, it can be verified that this will be the case if the volatility of the value of the portfolio is less than about 15%.) A bank can offer clients a $1,000 investment opportunity consisting of:

1. A 3-year zero-coupon bond with a principal of $1,000
2. A 3-year at-the-money European call option on the stock portfolio.

If the value of the portfolio increases the investor gets whatever $1,000 invested in the portfolio would have grown to. (This is because the zero-coupon bond pays off $1,000 and this equals the strike price of the option.) If the value of the portfolio goes down, the option has no value, but payoff from the zero-coupon bond ensures that the investor receives the original $1,000 principal invested.

-
- The attraction of a principal-protected note is that an investor is able to take a risky position without risking any principal.
    - the worst that can happen, is that he won't have any interest / dividend
- A bank will always build in a profit for itself when it creates a principal-protected note
- The bank can add value for the investor while making a profit itself.
    - The price of this investment strategy depends on
        - risk-free interest rates
        - volatility of the risky asset.

- A bank can however make it profitable again by
    - increasing the strike price of the option
    - increasing its life-time
    - It depends on the dividend yield, the higher it is the more profitable the product is for the bank

11.2 Strategies involving a single option and a stock

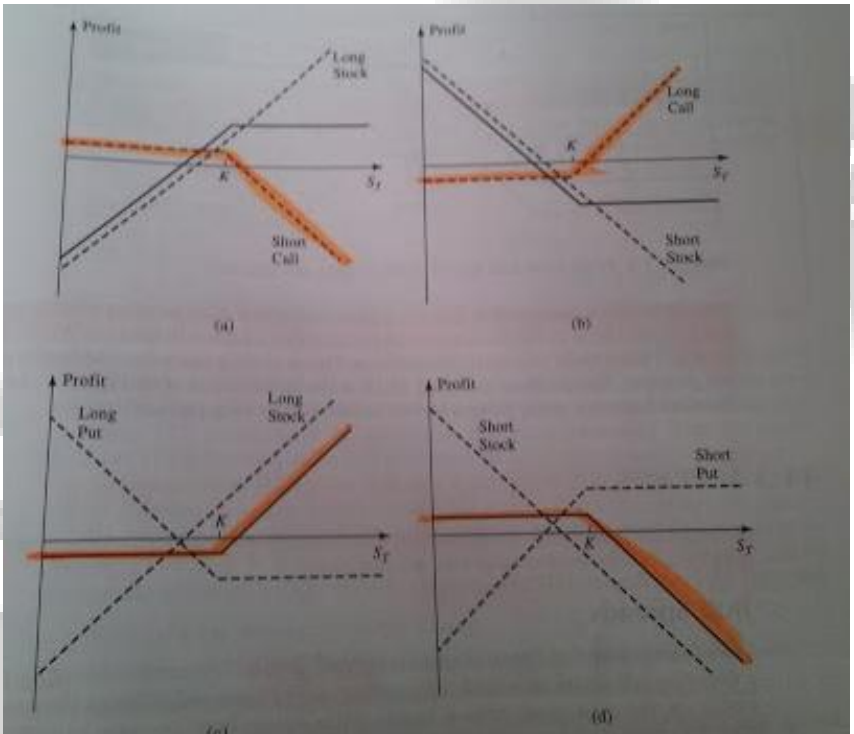


Figure 11.1 Profit patterns (a) long position in a stock combined with short position in a call; (b) short position in a stock combined with long position in a call; (c) long position in a put combined with long position in a stock; (d) short position in a put combined with short position in a stock

-
- (Dashed line: relationship between stock price and profit for individual securities)
- (Solid line: relationship between profit and stock price for whole portfolio)
- *Writing covering call (left upper graph)*: consists of a long position in a stock + a short position in a European call option. The long stock position covers the investor from the payoff on the short call, if there is a sharp rise in the stock price. → profit is the premium that you acquired by shorting the calls.
- *Reverse of writing a covered call (right upper graph)*: You short the stock and take a long positio.n call.
- *Protective put (left lower graph)*: long position in a put combined with long position in a stock.
- Put-call parity = $p + S_0 = c + Ke^{-rT} + D$

- p = price of the European put
- S_0 = the stock price
- c = the price of a European call
- K = strike price of both call and put
- r = risk-free rate
- T = the time of maturity of both call and put
- D = is the present value of the dividends anticipated during the life of the options

- A long position in a stock combined with a short position in a European call is equivalent to a short European put position plus a certain amount (= $Ke^{-rT} + D$)

11.3 Spreads

- A spread trading strategy involves taking a position in two or more options of the same type (i.e. two or more calls or two or more puts)
- *Bull spread:* by buying a European call option on a stock with a certain strike price and selling a European call option on the same stock with a higher strike price.
    - Investors in bull spread strategies hope that the stock price will increase
    - Both options have the same expiration date
    - profit:
    - Because a call price always decreases as the strike price increases, the value of the option sold is always less than the value of the option bought. A bull spread, created from calls, therefore requires an initial investment.
    - A bull spread strategy limits the investor's upside as well as downside risk.

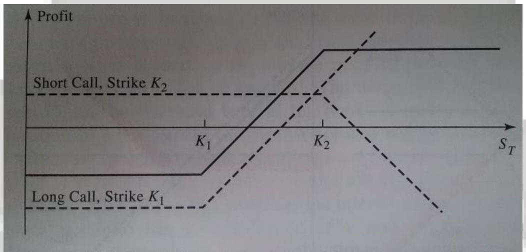


Figure 11.2 Profit from bull spread created using call options

    -
    - There are three types of bull spread
        - Both calls are initially out of the money
        - One call is initially in the money; the other call is initially out of the money
        - Both calls are initially in the money
    - Bull spreads can also be created by buying a European put with a low strike price and selling a European put with a high strike price.

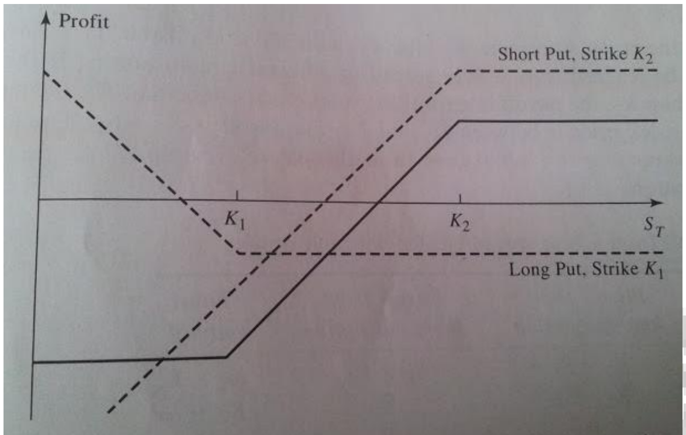


Figure 11.3 Profit from bull spread created using put options

- *Bear spreads*: buying a European put with one strike and selling a European put with another strike price.
  - Investors in bear spread hope that the stock prices will decrease.
  - The strike price of the option purchased is greater than the strike price of the option sold.

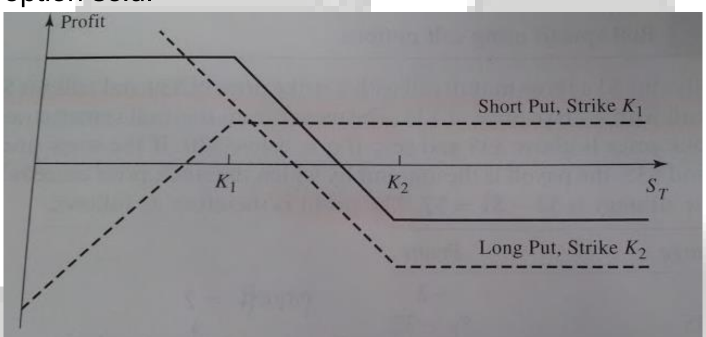


Figure 11.4 Profit from bear spread created using put options

Table 11.2 Payoff from a bear spread created with put options

Stock price range	*Payoff from long put option*	*Payoff from short put option*	*Total payoff*
$S_T \leqslant K_1$	$K_2 - S_T$	$S_T - K_1$	$K_2 - K_1$
$K_1 < S_T < K_2$	$K_2 - S_T$	0	$K_2 - S_T$
$S_T \geqslant K_2$	0	0	0

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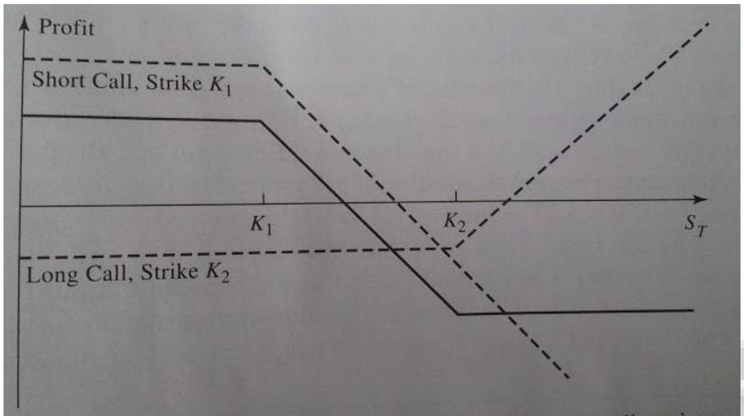

Figure 11.5 Profit from bear spread created using call options

- Like bull spreads, bear spreads limit both the upside profit and the downside risk.
- Bear spreads can also be created with calls instead of puts.

- Box spread
    - *Box spread*: is a combination of a bull call spread with prices K_1 and K_2 and a bear put spread with the same two strike prices.

Table 11.3 Payoff from a box spread

Stock price range	*Payoff from bull call spread*	*Payoff from bear put spread*	*Total payoff*
$S_T \leqslant K_1$	0	$K_2 - K_1$	$K_2 - K_1$
$K_1 < S_T < K_2$	$S_T - K_1$	$K_2 - S_T$	$K_2 - K_1$
$S_T \geqslant K_2$	$K_2 - K_1$	0	$K_2 - K_1$

    -
    - The value of a box spread is therefore always the present value of K_2 - K_1 or $(K_2 - K_1)e^{-rT}$ → if it is different then there is opportunity for arbitrage
    - Box spread arbitrage only works with European options.
- Butterfly spread
    - *Butterfly spread*: involves positions in options with three different strike prices.
    - it can be created by
        - buying European call option with relatively low strike price K_1
        - buying European call option with relatively high strike price K_3
        - Selling two European call option with strike price K_2, midway K_1 and K_3
    - This strategy leads to profit is the stock price stays close to K_2, but gives rise to a small loss if there is a significant stock price move in either direction.
    - The use of put options results in exactly the same spread as the use of call options.

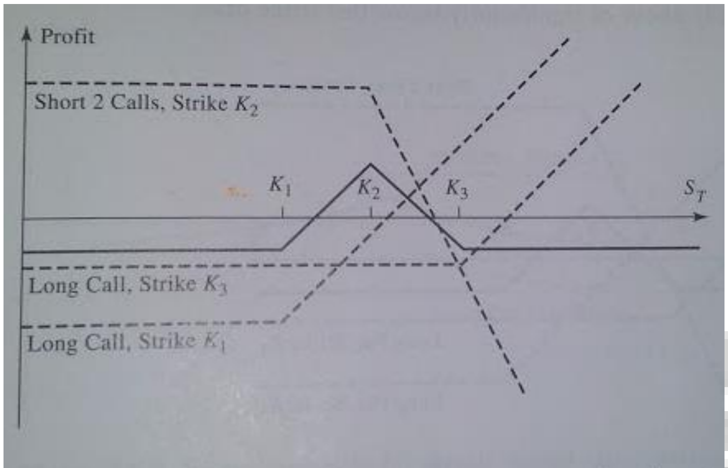

Figure 11.6 Profit from butterfly spread using call options

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Table 11.4 Payoff from a butterfly spread

Stock price range	*Payoff from first long call*	*Payoff from second long call*	*Payoff from short calls*	*Total payoff**
$S_T \leqslant K_1$	0	0	0	0
$K_1 < S_T \leqslant K_2$	$S_T - K_1$	0	0	$S_T - K_1$
$K_2 < S_T < K_3$	$S_T - K_1$	0	$-2(S_T - K_2)$	$K_3 - S_T$
$S_T \geqslant K_3$	$S_T - K_1$	$S_T - K_3$	$-2(S_T - K_2)$	0

* These payoffs are calculated using the relationship $K_2 = 0.5(K_1 + K_3)$.

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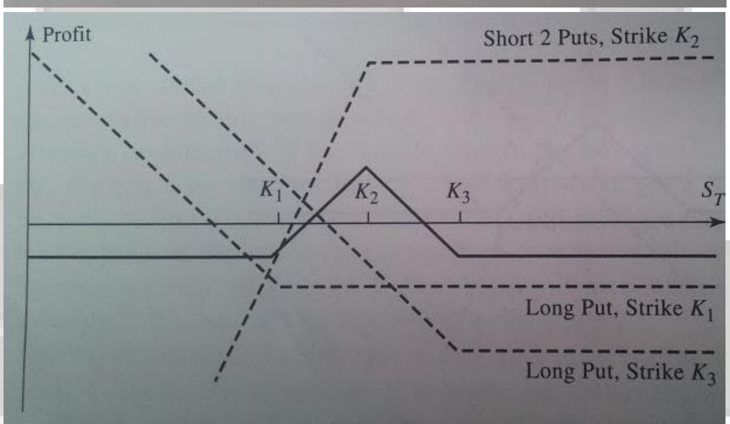

Figure 11.7 Profit from butterfly spread using put options

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- Calendar spread
  - *Calendar spread*: can be created by selling a European call option with a certain strike price and buying a longer-maturity European call option with the same strike price.
  - The longer the maturity of an option, the more expensive it is.
  - The pattern is similar to the profit from the butterfly spread.
  - The investor makes a profit if the stock price at expiration of the short-term maturity is close to the strike price of the short-maturity option.

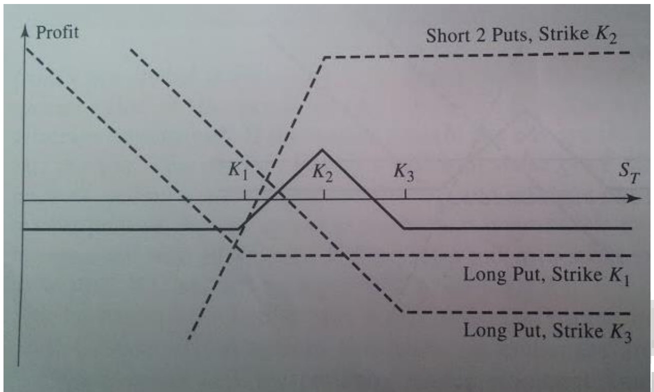

Figure 11.7 Profit from butterfly spread using put options

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- Straddle
  - *Straddle*: buying European call and put with the same strike price and expiration date.
  - If there is a sufficiently large move in either direction, a significant profit will result.

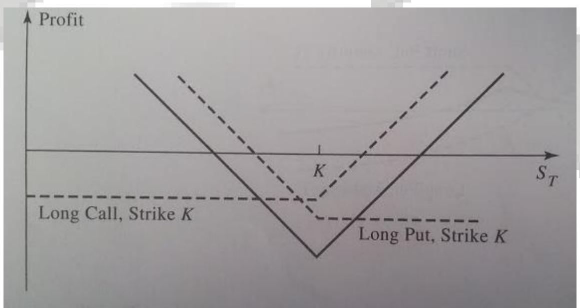

Figure 11.10 Profit from a straddle

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Table 11.5 Payoff from a straddle

Range of stock price	*Payoff from call*	*Payoff from put*	*Total payoff*
$S_T \leqslant K$	0	$K - S_T$	$K - S_T$
$S_T > K$	$S_T - K$	0	$S_T - K$

-

- *Strip*: a long position in one European call and two European puts with the same strike and expiration date → betting decrease in stock price is more likely then increase.
- *Strap:* a long position in two European calls and one European put with the same strike and expiration date.→ betting increase in stock price is more likely then decrease.

- *Strangle*: Investor buys a European put and a European call with the same expiration date and different strike prices.
    - A strangle is a similar strategy to a straddle

- 

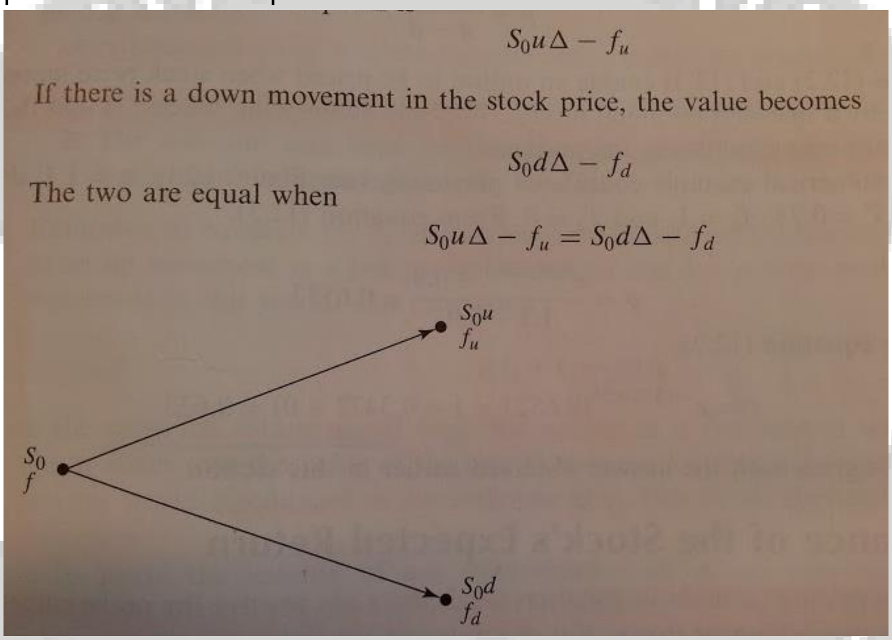
-

Range of	Payoff from	Payoff from	Total

Chapter 12: Introduction to binomial trees

12.1 A one-step binomial model and a no-arbitrage argument

- To use a binomial model there is one assumption:
  - There is no arbitrage opportunity.
- We set up a portfolio of the stock and the option in such a way that there is no uncertainty about the value of the portfolio at the end of three months. We then argue that because the portfolio has no risk, the return must equal the risk-free interest rate
  - e.g consider a portfolio consisting of a long position in Δ stock and a short position in one call option.



$$S_0 u\Delta - f_u$$

If there is a down movement in the stock price, the value becomes

$$S_0 d\Delta - f_d$$

The two are equal when

$$S_0 u\Delta - f_u = S_0 d\Delta - f_d$$

  -
  - And the following situation where stock price can move from S_0 to upwards S_0u or downwards S_0d
  - If the stock price go up we suppose the payoff from the option is f_u
  - If the stock price goes down we suppose the payoff is f_d
  - We calculate the value of Δ that makes the portfolio riskless.
  - If there is a upward stock price → $S_0u\Delta - f_u$
  - If there is a downward stock price → $S_0d\Delta - f_d$
  - thus this means that $S_0u\Delta - f_u = S_0d\Delta - f_d$
  - or $\Delta = \frac{f_u - f_d}{S_0 u - S_0 d}$
  - The present value of the portfolio is $(S_0 u\Delta - f_u)e^{-rT}$
  - $f = e^{-rT}[p\, f_u + (1-p)\, f_d]$
  - $p = \frac{e^{rT} - d}{u - d}$ → should be interpret as an upward movement in a risk-neutral world

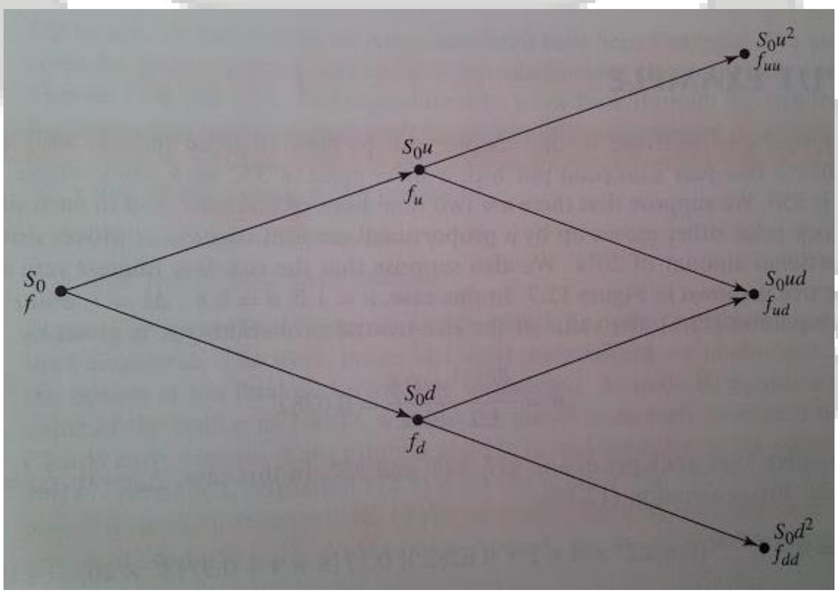
  - with u = S_u / S_0
  - with d = S_d / S_0
- The only assumption: There is no arbitrage opportunities in the market
- The probabilities of future up or down movements are already incorporated into the price of the stock

12.2 Risk-Neutral Valuation

- *Risk-neutral valuation*: this means that investors do not increase the expected return they require from an investment to compensate for increased risk.
- When we are pricing an option in terms of the price of the underlying stock, risk preferences are unimportant.
- A risk-neutral world simplifies the pricing of derivatives
  - The expected return on a stock (or any other investment is the risk-free rate
  - The discount rate used for the expected payoff on an option (or any other instrument) is the risk-free rate.
- expected stock price = $E(S_T) = S_0 e^{rT}$
- Risk neutral states, when we assume the world is risk-neutral, we get the right price for a derivative in all worlds, not just in a risk-neutral one
- Risk-neutral valuation gives the same answer as no-arbitrage arguments
- *p* is the probability of an up movement in a risk-neutral world

12.3 Two-step binomial trees

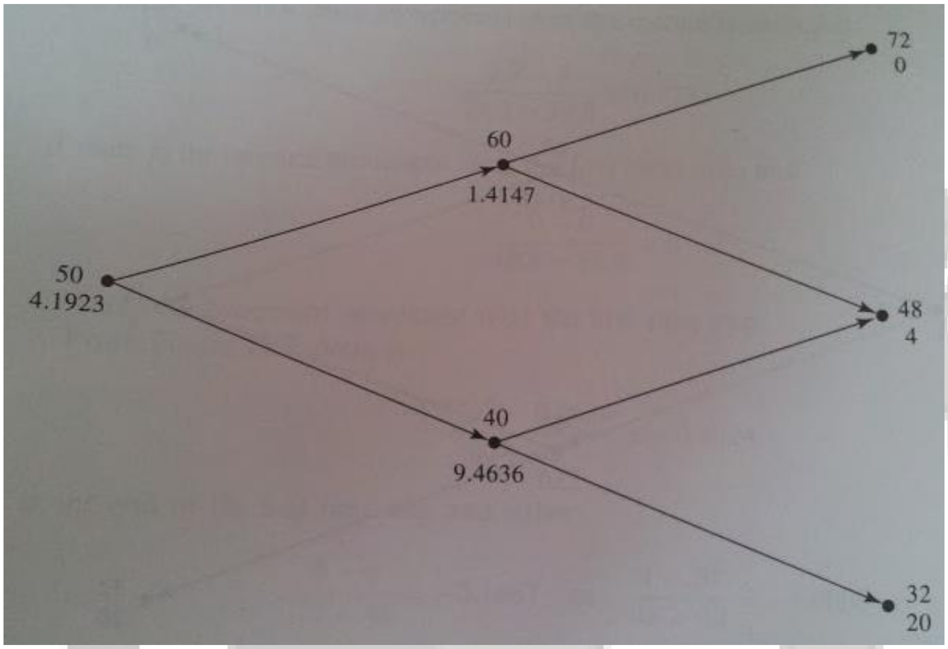
- The objective of multiple step binomial trees is to calculate the option price at the initial node of the tree.



-
- $f = e^{-r\Delta t}[p f_u + (1 - p) f_d]$
- $p = \dfrac{e^{r\Delta t} - d}{u - d}$
- $f_u = e^{-r\Delta t}[p f_{uu} + (1 - p) f_{ud}]$

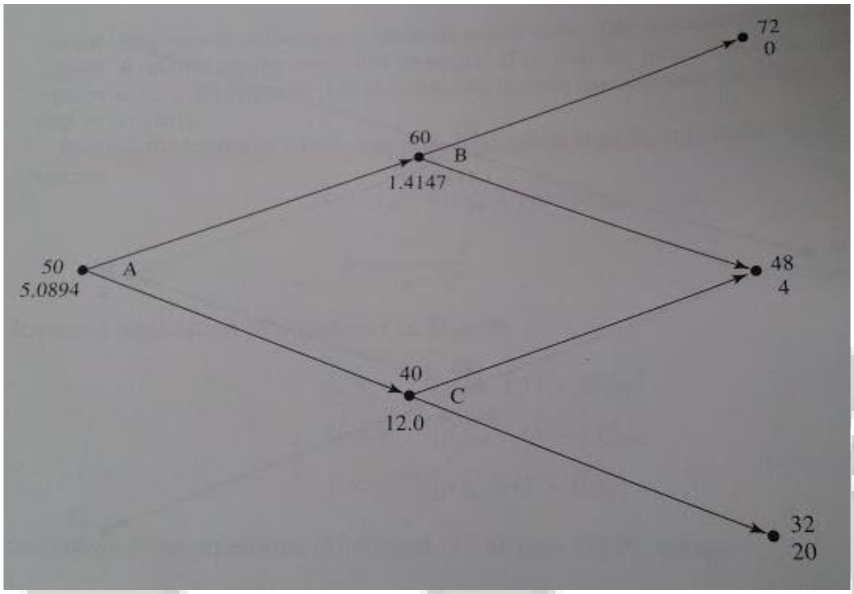
- $f_d = e^{-r\Delta t}[p\, f_{ud} + (1 - p)\, f_{dd}]$
- $f = e^{-r\Delta t}[p\, f_u + (1 - p)\, f_d]$

12.4 A put example



12.5 American Options

- The same tree can be used for american options as well.
- The procedure is to work back through the tree from the end to the beginning, testing at each node to see whether early exercise is optimal
- The value of the option at the final node is the same as for European options. At earlier nodes the value of the option is greater of:
  - the value given by $f = e^{-r\Delta t}[p\, f_u + (1 - p)\, f_d]$
  - The payoff from early exercise

- 

12.6 Delta

- The delta Δ of a stock option is the ratio of the change in the price of the stock option to the change in the price of the underlying stock.
- It is the number of units of the stock we should hold for each option shorted in order to create a riskless portfolio. → Delta hedging
- Delta of call option is positive
- Delta of put option is negative.
- Delta changes over time → thus we need to adjust the holdings in stocks periodically.

12.7 Determining u and d.

- In practice u and d are determined from the stock price volatility, σ
  - $u = e^{\sigma\sqrt{\Delta t}}$
  - $d = \frac{1}{u}$
  - Δt= the length of one time step on the tree
- $p = \frac{a-d}{u-d}$ with $a = e^{r\Delta t}$

12.8 Increasing the number of time steps

- as the number of time steps is increased (so that Δt becomes smaller), the binomial models makes the same assumptions about stock price behaviour as the Black-Scholes-Merton model

12.10 Options on other assets

- For other options we can construct and use binomial trees in exactly the same way as for options on stocks except that the equations for p change

Chapter 13: Valuing stock options: the Black-Scholes-Model

13.1 assumptions about how stock prices evolve

- The Black-Scholes model considers a non-dividend-paying stock and assumes that the return on the stock in a period of time is normally distributed.
- When the difference in time is small, the percentage change in the stock price is normal with standard deviation of the $volatility * \sqrt{difference\ in\ time}$
- When longer time periods are considered, it is necessary to be more precise about the future stock price distribution.
- The assumption implies that the stock price at any future time has a lognormal distribution.

13.2 Expected return

- The expected return, μ required by investors from a stock depends on the riskiness of the stock, and on the level of interest rates in the economy
    - the higher the riskiness, the higher the expected return required
    - the higher the level of interest, the higher the expected return required

13.3 Volatility

- The volatility of a stock, σ, is a measure of our uncertainty about the returns provided by the stock.
- The volatility of a stock can be defined as the standard deviation of the return provided by the stock in one year when the return is expressed using continuous compounding.

13.4 Estimating volatility from historical data

- The volatility per annum is calculated from the volatility per trading day using the formula
    - $volatility\ per\ annum = volatility\ per\ trading\ day * \sqrt{Number\ of\ trading\ days\ per\ annum}$
- The life of an option is also usually measured using trading days rather than calendar days. It is calculated as T years,
    - $T = \frac{Number\ of\ trading\ days\ until\ option\ maturity}{252}$

13.5 Assumptions underlying Black-Scholes-Merton

1. Stock price behaviour corresponds to the lognormal model with μ and σ constant
2. There are no transactions costs or taxes. All securities are perfectly divisible
3. There are no dividends on the stock during the life of the option.
4. There are no riskless arbitrage opportunities
5. Security trading is continuous
6. Investors can borrow or lend at the same risk-free rate of interest.

7. The short-term risk-free rate of interest, *r*, is constant.

13.6 The key no-arbitrage argument

- The arguments that can be used to price options are analogous to the no-arbitrage arguments when stock price changes were assumed to be binomial.
- In the absence of arbitrage opportunities, the return from a riskless portfolio must be the risk-free rate, *r*.
- the reason a riskless portfolio can be set up is that the stock price and the option price are both affected by the same underlying source of uncertainty: stock price movements
    - in short time period, the price of call option is positively correlated with the price of the underlying stock
    - the price of a put option is negatively correlated with the price of the underlying stock

13.7 The Black-Scholes-Merton pricing formulas

- The Black-Scholes-Merton formulas for the prices of European calls and Puts on non-dividend paying stocks are
    - $c = S_0 N(d_1) - Ke^{-rT} N(d_2)$
    - $p = Ke^{-rT} N(-d_2) - S_0 N(-d_1)$
    - $d_1 = \frac{ln(S_0 / K) + (r + \sigma^2/2) * T}{\sigma\sqrt{T}}$
    - $d_2 = \frac{ln(S_0 / K) + (r - \sigma^2/2) * T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$
        - c = european call price
        - p = european put price
        - S_0 = current stock price
        - K = strike price
        - r = risk-free rate
        - T = time to expiration
        - σ = volatility of stock price for a non-dividend paying stock
- The function N(x) is the cumulative probability function for a standardized normal variable.
    - it is the probability that a variable with a standard normal distribution will be less than x.
- The American call price, C, equals the european call price, c
- There is no exact analytical formula to value an American put
- In theory, the Black-Scholes formula correct only if the short-term interest rate is constant. But in practice it is used for the *r* of time *T*
- Properties of the Black-Scholes model
    - When the stock price S_0 becomes very large
        - call option will almost certainly be exercised thus call price = $S_0 - Ke^{-rT}$
            - This is because d_1 and d_2 will become large and $N(d_1)$ $N(d_2)$ will become 1
        - price of the put option approaches zero

        - This because $N(-d_1)$ and $N(-d_2)$ are both close to zero when S_0 is large
    - When the stock price becomes very small
        - price of call option will approach zero
            - This because $N(d_1)$ and $N(d_2)$ are then both close to zero
        - Price of the put option is close to $Ke^{-rT} - S_0$
            - This because $N(-d_1)$ and $N(-d_2)$ become close to 1
- $N(d_2)$ has a simple interpretation: It is the probability that a call option will be exercised in a risk-neutral world.

13.8 Risk-Neutral valuation

- Risk-Neutral valuation = Any security dependent on other traded securities can be valued on the assumption that investors are risk neutral.
    - This does not state that investors are risk neutral
    - it state that derivatives such as options can be valued on the assumption that investors are risk neutral
    - it means that investors' risk preferences have no effect on the value of the stock option when it is expresses as a function of the price of the of the underlying stock.
    - It explains why the put and call formulas of the Black-Scholes does not involve the stock's expected return, μ
- Risk-neutral valuation is a very powerful tool because in a risk-neutral world two simple results hold.
    - The expected return from all investment assets it the risk-free rates.
    - The risk-free interest rate is the appropriate discount rate to apply to any expected future cash flow
- Options and other derivatives can be valued using risk-neutral valuation. The procedure is as follow
    1. Assume that the expected return from the underlying asset is the risk free rate thus $\mu = r$
    2. Calculate the expected payoff
    3. Discount the expected payoff at the risk-free interest rate
- This procedure can be used to value a forward contract on a non-dividend paying stock
    - e.g. consider a long forward contract that matures at *T* with strike price *K*
    - the value at maturity of the contract is $S_T - K$
    - The expected value of S_T is $S_0e^{\mu T}$
    - In a risk neutral world, it becomes S_0e^{rT}
    - The expected payoff is therefore $S_0e^{rT} - K$
    - Discounting at the risk-free rate r for time T gives the value *f*, of the forward contract as $f = e^{-rT}(S_0e^{rT} - K) = S_0 - Ke^{-rT}$

13.9 Implied volatility

- Volatility is the only parameter that cannot be observed directly in the Black-Scholes model
- Therefore, we use the implied volatility
- *implied volatility:* This is the volatility implied by an option price observed in the market
- It is not possible to invert the Black Scholes model so that the volatility is expressed as a function.
- You can only find implied volatility using iterative search procedure (trial and error)
- Implied volatility can be used to monitor the market's opinion about the volatility of a particular stock.
- Historical volatilities are backward looking
- Implied volatilities are forward looking.
- The SPX VIX is an index of the implied volatility of 30-day options on the S&P 500, and widely used for trading

13.10 Dividends

Example 13.6 Using Black–Scholes–Merton when there are dividends

Consider a European call option on a stock with ex-dividend dates in two months and five months. The dividend on each ex-dividend date is expected to be \$0.50. The current share price is \$40, the exercise price is \$40, the stock price volatility is 30% per annum, the risk-free rate of interest is 9% per annum, and the time to maturity is six months. The present value of the dividends is

$$0.5e^{-0.09\times 2/12} + 0.5e^{-0.09\times 5/12} = 0.9741$$

The option price can therefore be calculated from the Black–Scholes–Merton formula with $S_0 = 40 - 0.9741 = 39.0259$, $K = 40$, $r = 0.09$, $\sigma = 0.3$, and $T = 0.5$:

$$d_1 = \frac{\ln(39.0259/40) + (0.09 + 0.3^2/2)\times 0.5}{0.3\sqrt{0.5}} = 0.2020$$

$$d_2 = \frac{\ln(39.0259/40) + (0.09 - 0.3^2/2)\times 0.5}{0.3\sqrt{0.5}} = -0.01012$$

Using the NORMDIST function in Excel gives

$$N(d_1) = 0.5800 \quad \text{and} \quad N(d_2) = 0.4959$$

and from equation (13.5) the call price is

$$39.0259 \times 0.5800 - 40e^{-0.09\times 0.5} \times 0.4959 = 3.67$$

or \$3.67.

- American call options
    - If there is no dividend than american call option should never be exercised early.
    - However, if there is dividend it is sometimes optimal to exercise early
    - The dividend, will make both the stock and the call option less valuable

        - Call options are most likely to be exercised early immediately before the final ex-dividend rate.
- Black's approximation: involves calculating the prices of two european options
    1. A European option that matures at the same time as the american option
    2. A European option maturing just before the latest ex-dividend date that occurs during the life of that option.
    - The American option is set equal to the higher of these two European option price.

Chapter 15 Options on Stock Indices and Currencies

15.1 Options on stock indices

- One index option contract is on 100 times the index
- Portfolio managers can use index options to limit their downside risk
- A beta of 1.0 implies that the returns from the portfolio mirror those from the index.

Example 15.1 Protecting the value of a portfolio that mirrors the S&P 500

A manager in charge of a portfolio worth $500,000 is concerned that the market might decline rapidly during the next three months and would like to use index options as a hedge against the portfolio declining below $450,000. The portfolio is expected to mirror closely the S&P 500, which is currently standing at 1,000.

The Strategy
The manager buys five put option contracts with a strike price of 900 on the S&P 500.

The Result
The index drops to 880.
The value of the portfolio drops to $440,000.
There is a payoff of $10,000 from the five put option contracts.

-
    - The value of the portfolio is protected against the possibility of the index falling below K if, for each $100S_0$ dollars in the portfolio, the managers buys one put option contract with strike price K.
    - The strike price of the contracts are 900 because he expects the value to drop from 500.000 to 450.000 which is a 10% decrease. Thus he hedges himself for 1000-10% = 900.
    - The payoff from the option will be 5 x (900 - 880) x 100 = $10.000 which increases the value of his portfolio up to 450.000
- If the portfolio's beta is not 1.0 then beta put options must be purchased for each $100S_0$ dollars in the portfolio where S_0 is the current value of the index.
- If we take the same example as above, but with a beta of 2.0 then the number of put options is equal to $2.0 * \frac{500.000}{1.000*100} = 10$contracts
- To calculate the appropriate strike price, the capital asset pricing model can be used.

- **Table 15.1** Calculation of expected value of portfolio when the index is 1,040 in three months and $\beta = 2.0$

Value of index in three months:	1,040
Return from change in index:	40/1,000, or 4% per three months
Dividends from index:	$0.25 \times 4 = 1\%$ per three months
Total return from index:	$4 + 1 = 5\%$ per three months
Risk-free interest rate:	$0.25 \times 12 = 3\%$ per three months
Excess return from index over risk-free interest rate:	$5 - 3 = 2\%$ per three months
Expected excess return from portfolio over risk-free interest rate:	$2 \times 2 = 4\%$ per three months
Expected return from portfolio:	$3 + 4 = 7\%$ per three months
Dividends from portfolio:	$0.25 \times 4 = 1\%$ per three months
Expected increase in value of portfolio:	$7 - 1 = 6\%$ per three months
Expected value of portfolio:	$500,000 × 1.06 = $530,000

- Capital Asset pricing model:

$$\overline{r_a} = r_f + \beta_a(\overline{r_m} - r_f)$$

Where:
r_f = Risk free rate
β_a = Beta of the security
$\overline{r_m}$ = Expected market return

- There are two reasons why the cost of hedging increases as the beta of a portfolio increases:
    - More put options are required
    - And a higher strike price is needed

15.2 Currency options

- Currency options are primarily traded in the over-the-counter market.
- For a corporation wishing to hedge a foreign exchange exposure, foreign currency options are an alternative to forward contracts.
- Whereas a forward contract locks in the exchange rate for a future transaction, an option provides a type of insurance (at a cost price).
- Range forward contracts
    - Is a variation on a standard forward contract for hedging forward exchange risk.
    - Short range forward contract (when you know you RECEIVE foreign currency): buy a European put with strike price K_1 and sell European call with a strike price K_2 with ''K_1 < expected exchange rate < K_2''
    - Long range forward contract (when you know you have to PAY foreign currency): sell a European put option with strike price K_1 and buy a European call option with strike price K_2

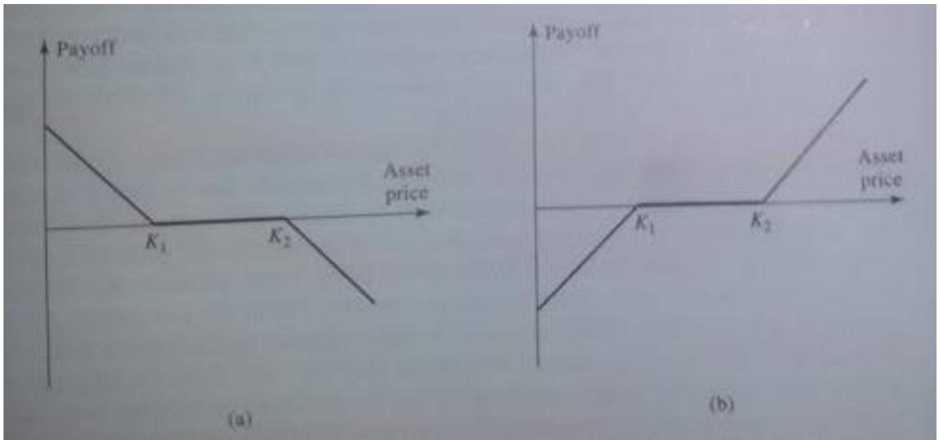


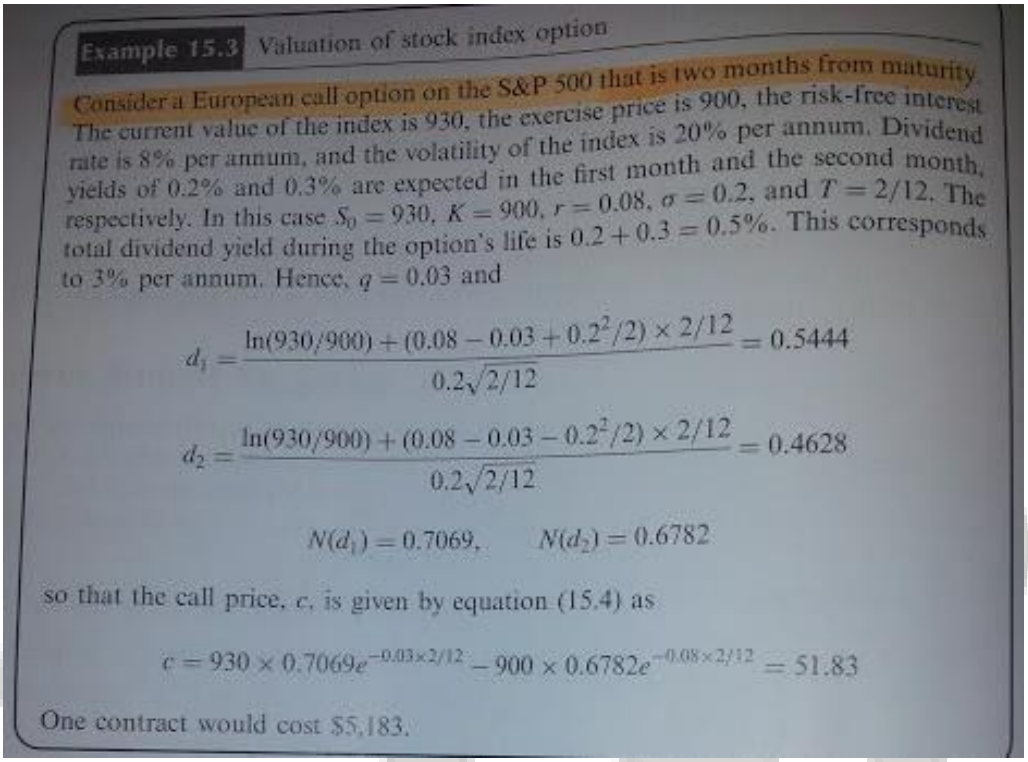
Figure 15.1 Payoffs from (a) short and (b) long range forward contract

- In practice, a range forward contract is set up so that the price of the put options equals the price of the call option → so it costs nothing to set up the contract.

15.3 Options on Stocks paying known dividend yields

- Dividends cause stock prices to be reduced on the ex-dividend rate by the amount of the dividend payment.
- The payment of a dividend yield at a rate *q* therefore causes the growth rate in the stock price to be less than it would otherwise.
- When valuing a European option lasting for time *T* on a stock paying a known dividend yield at rate *q*, we reduce the current stock price from S_0 to S_0e^{-qT} and then value the option as though the stock pays no dividends.
- Lower bound for option prices
  - Lower bound for the European call option price *c*
    - $c \geqslant \max(S_0e^{-qT} - Ke^{-rT}, 0)$
  - Lower bound for the European put option price *p*
    - $p \geqslant \max(Ke^{-rT} - S_0e^{-qT}, 0)$
- Put-Call parity with dividend yield
  - $c + Ke^{-rT} = p + S_0e^{-qT}$
  - for american options it becomes
  - $S_0e^{-qT} - K \leqslant C - P \leqslant S_0 - Ke^{-rT}$
- $c = S_0e^{-qT}N(d_1) - Ke^{-rT}N(d_2)$
- $p = Ke^{-rT}N(-d2) - S_0e^{-qT}N(-d_1)$
- with $d_1 = \frac{ln(S_0/K) + (r - q + \sigma^2 / 2)\,T}{\sigma\sqrt{T}}$

15.4 Valuation of european stock index options



Example 15.3 Valuation of stock index option

Consider a European call option on the S&P 500 that is two months from maturity. The current value of the index is 930, the exercise price is 900, the risk-free interest rate is 8% per annum, and the volatility of the index is 20% per annum. Dividend yields of 0.2% and 0.3% are expected in the first month and the second month, respectively. In this case $S_0 = 930$, $K = 900$, $r = 0.08$, $\sigma = 0.2$, and $T = 2/12$. The total dividend yield during the option's life is $0.2 + 0.3 = 0.5\%$. This corresponds to 3% per annum. Hence, $q = 0.03$ and

$$d_1 = \frac{\ln(930/900) + (0.08 - 0.03 + 0.2^2/2) \times 2/12}{0.2\sqrt{2/12}} = 0.5444$$

$$d_2 = \frac{\ln(930/900) + (0.08 - 0.03 - 0.2^2/2) \times 2/12}{0.2\sqrt{2/12}} = 0.4628$$

$$N(d_1) = 0.7069, \qquad N(d_2) = 0.6782$$

so that the call price, c, is given by equation (15.4) as

$$c = 930 \times 0.7069e^{-0.03\times2/12} - 900 \times 0.6782e^{-0.08\times2/12} = 51.83$$

One contract would cost $5,183.

-
- we assumed that index could be treated as an asset paying a known yield
- Thus we can use the dividend yielding formulas to value European options.
- The calculation of *q* should include only dividends for which the ex-dividend dates occur during the life of the option
- You can also determine the value of forward prices by substituting them in the formula.
- $c = F_0 e^{-rT} N(d_1) - K e^{-rT} N(d_2)$
- $p = K e^{-rT} N(-d2) - F_0 e^{-rT} N(-d_1)$
- with $d_1 = \frac{ln(F_0/K) + \sigma^2 T/2}{\sigma\sqrt{T}}$

15.5 Valuation of European currency options
- To value currency options, we define S_0 as the spot exchange rate
- $c = S_0 e^{-r_f T} N(d_1) - K e^{-rT} N(d_2)$
- $p = K e^{-rT} N(-d2) - S_0 e^{-r_f T} N(-d_1)$
- with $d_1 = \frac{ln(F_0/K) + (r - r_f + \sigma^2 / 2) T}{\sigma\sqrt{T}}$ with r_f the risk free rate in the foreign currency.

15.6 American options
- for a non dividend-paying stock, the probability of an up movement is
  - $p = \frac{a - d}{u - d}$
  - with $a = e^{r\Delta t}$
- for options on indices and currencies, the formula for *p* is the same but *a* is defined differently

- $a = e^{(r-q)\Delta t}$ for an index with q being the dividend yield
- $a = e^{(r-r_f)\Delta t}$for an option on a currency with r_f the foreign risk-free rate.

- In some circumstances it is optimal to exercise American currency and index options prior to maturity. Thus, American currency and index options are worth more than their European counterparts.
    - Call options on high-interest currencies and put options on low-interest currencies are the most likely to be exercises prior to maturity
    - Call options on indices with high dividend yields and put options on indices with low dividend yields are most likely to be exercised early.

The course "Project Financial Instruments" covers some important topics in financial products, such as futures, forwards, options, SWAPs, CDO. This course will try to obtain a balance between theories and empirical works. After following this course you should be able to:
• understand the functioning and the pricing of futures, forwards, options, SWAPS.
• gain insights on 2008 financial crisis, in particular the financial products that lead to the crisis.
• perform risk analysis of multinational companies.
• organize your analysis in a paper and explain it to others in a presentation.

Tips en opmerkingen over de samenvatting kunnen naar secretaris@esvnijmegen.nl gestuurd worden.